

A New Differential Modulation for Coded OFDM With Multiple Transmit Antennas

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Abstract—A new differential modulation scheme is presented for coded orthogonal frequency-division multiplexing (OFDM) systems with multiple transmit antennas in frequency- and time-selective channels. In contrast to an earlier scheme that involves differential modulation in space and time (ST) across two code matrices, the new scheme performs it in space and frequency (SF) within only one code matrix. As such, the shortest coherence time of the channel that can be handled is reduced, which makes the differential SF scheme more resistant to fast fading. Along with a suitable spectral encoder and interleaver, both the differential ST and SF schemes offer joint spatio-spectral diversity and coding gain. Numerical simulation confirms that SF is indeed more resistant to time-selective fading than ST; however, the former also suffers some performance loss caused by frequency-selective fading and is preferred only when fast fading is prevalent.

Index Terms—Coded orthogonal frequency-division multiplexing (OFDM), differential modulation, frequency- and time-selective fading, spatio-spectral diversity.

I. INTRODUCTION

DIFFERENTIAL space-time modulation, offering diversity and coding gain without channel estimation for wireless systems equipped with multiple transmit antennas, has received considerable interest in recent years (e.g., [1]–[7]). While earlier efforts were primarily focused on narrowband systems in flat fading channels and the goal was to provide spatial (transmit) diversity (e.g., [1]–[3]), there have been several recent developments of coded differential modulation schemes that offer joint spatio-spectral diversity for orthogonal frequency-division multiplexing (OFDM)-based broadband systems in frequency-selective channels (e.g., [5]–[7]).

A commonality shared by the above broadband solutions is that differential modulation is performed over two space-time-frequency (STF) code matrices, and each STF matrix can be thought of as a “super-symbol” that is transmitted using multiple time slots (a *time slot* refers to an OFDM symbol duration). For example, with two transmit antennas, each STF code matrix covers two time slots, and for acceptable performance, the schemes of [5]–[7] would require the channel to remain

approximately unchanged over two STF matrices or four time slots. Since the size of the STF matrix grows with the number of transmit antennas, the period of time over which the channel should remain approximately static increases as more transmit antennas are involved. This was considered a limitation of the above broadband differential solutions.

The above discussion points to the need for new broadband differential solutions that can be encoded and decoded within a shorter period of time. In this letter, we present one such solution that can be considered as a modification of the scheme in [5]. Specifically, the new scheme performs differential modulation in space and frequency (SF) across two adjacent subcarriers but within one STF code matrix, whereas [5] involves differential modulation in space and time (ST) over two STF code matrices. The premise of the new SF scheme is that the channel frequency response over two adjacent subcarriers is approximately the same to admit differential demodulation using the signals received on the two subcarriers. For channels with a large delay spread, the frequency response may vary considerably over adjacent subcarriers, which can cause some performance loss of the SF scheme. Hence, there is a tradeoff in handling time-selective and frequency-selective fading.

During the review of this letter, we were informed of [8], which contains a single-block differential modulation scheme that can be thought of as an extension of [7]. For a given transmission rate, [8] incurs larger constellation expansion, which is generally undesirable (e.g., loss in coding gain and increased decoding complexity). A similar observation was made in [6] between the scheme of [5] and that of [7].

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively; \mathbf{I}_M denotes the $M \times M$ identity matrix; $\text{diag}\{\cdot\}$ denotes a diagonal matrix; $\|\cdot\|$ denotes the vector two-norm; and finally, \otimes denotes the matrix/vector Kronecker product.

II. SYSTEM DESCRIPTION

For brevity, we consider only the case with two transmit antennas and one receive antenna. Extension to systems with an arbitrary number of transmit/receive antennas can be made in a way similar to that in [6]. Fig. 1 depicts the diagram of the proposed system. Specifically, at the transmitter, a group of information bits are first encoded and interleaved in the frequency domain (see Section III-A for details) to form two $P \times 1$ vectors $\mathbf{s}(2n)$ and $\mathbf{s}(2n+1)$, $n = 0, 1, \dots$, where P is the number of subcarriers, and the coded symbols are drawn from a phase-shift keying (PSK) constellation to facilitate differential modulation. The spectrally coded vectors $\mathbf{s}(2n)$ and

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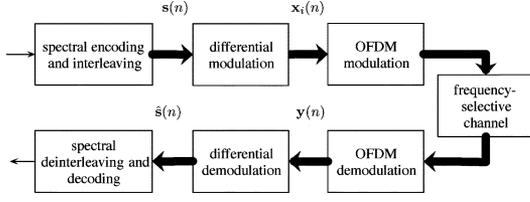


Fig. 1. Block diagram of the proposed system. Thick arrows indicate multi-channel (vector) signal flows across adjacent blocks.

$\mathbf{s}(2n+1)$ are differentially encoded and mapped to a $2P \times 2$ STF matrix (see Section III-B)

$$\mathbf{X}(n) \triangleq \begin{bmatrix} \mathbf{x}_1(2n) & \mathbf{x}_1(2n+1) \\ \mathbf{x}_2(2n) & \mathbf{x}_2(2n+1) \end{bmatrix}. \quad (1)$$

Next, $\mathbf{x}_i(2n+k)$

, for $i = 1, 2$ and $k = 0, 1$, is OFDM modulated, and the resulting OFDM symbol is transmitted from the i th antenna during the $(2n+k)$ th symbol interval.

At the receiver, the received signal is first demodulated via an OFDM demodulator, whose outputs are used to form two $P \times 1$ vectors $\mathbf{y}(2n)$ and $\mathbf{y}(2n+1)$, which correspond to the transmission of $\mathbf{X}(n)$ over two OFDM symbol intervals. Differential demodulation is next applied on $\mathbf{y}(2n)$ and $\mathbf{y}(2n+1)$ to yield $\hat{\mathbf{s}}(2n)$ and $\hat{\mathbf{s}}(2n+1)$, which are next spectrally deinterleaved and decoded to recover the information bits.

The baseband equivalent channel between the i th transmit antenna and the receive antenna is modeled as a finite impulse response (FIR) filter with coefficients $\{h_i(l)\}_{l=0}^L$, where L denotes the channel order. Note that the channel is assumed invariant for ease of presentation; however, the proposed scheme is tested in time-varying channels in Section IV. Due to OFDM modulation, the frequency-selective channel is equivalent to P parallel frequency-flat subchannels, with the frequency response for the p th subchannel given by $H_i(p) = \sum_{l=0}^L h_i(l) \exp(-j2\pi lp/P)$. The received signal after OFDM demodulation can be expressed as

$$y(2n+k;p) = \sum_{i=1}^2 H_i(p)x_i(2n+k;p) + w(2n+k;p) \quad k = 0, 1; p = 0, 1, \dots, P-1 \quad (2)$$

where $y(2n+k;p)$ and $x_i(2n+k;p)$ denote the p th element (i.e., the p th subcarrier) of $\mathbf{y}(2n+k)$ and $\mathbf{x}_i(2n+k)$, respectively, and $w(2n+k;p)$ denotes the zero-mean complex white Gaussian noise with variance $N_0/2$ per dimension.

III. PROPOSED SCHEME

Although our primary focus is the differential modulator and demodulator in Fig. 1, they cannot provide joint spatio-spectral diversity without a suitable spectral encoder and interleaver. In the following, we first briefly discuss the spectral encoder and interleaver before we present the new differential modulation scheme.

A. Spectral Encoding and Interleaving

A system with two transmit antennas and one receive antenna in a frequency-selective Rayleigh fading channels with $L+1$ paths can provide up to $2(L+1)$ spatio-spectral diversity [6]. Any code with a minimum Hamming distance no less than $L+1$ can be used for spectral encoding in Fig. 1 to achieve the full spatio-spectral diversity [6]. One class of codes that meets the condition is the so-called linear constellation decimation (LCD) code used in [5] and [6]. In particular, LCD codes are length- $(L+1)$ non-binary block codes with minimum Hamming distance $L+1$, and they are obtained by linearly decimating a PSK constellation. To facilitate comparison with the differential ST scheme [5], we use LCD codes in the following for spectral encoding.

For ease of presentation, we assume P is an integer multiple of $L+1$

$$M \triangleq P/(L+1). \quad (3)$$

The P subcarriers are divided into M groups, each containing $L+1$ subcarriers to carry one LCD codeword. The LCD spectral encoder works in a batch processing fashion. For each epoch of processing, it takes in a sequence of information bits and maps them into $2M$ LCD codewords (see [6] for details of the encoding process): $\mathbf{s}_m(2n+k)$, $m = 0, 1, \dots, M-1$, and $k = 0, 1$. These LCD codewords are next used to form $\mathbf{s}(2n+k)$ as follows:

$$\mathbf{s}(2n+k; m : M : m+LM) \triangleq \mathbf{s}_m(2n+k) \quad (4)$$

where we use the Matlab column notation to denote that the m th, $(m+M)$ th, \dots , $(m+LM)$ th elements of $\mathbf{s}(2n+k)$ are formed from the length- $(L+1)$ codeword $\mathbf{s}_m(2n+k)$. Effectively, this implements a uniform interleaving of the coded symbols, which is critical in achieving joint spatio-spectral diversity and coding gain in a Rayleigh fading environment [6].

B. Differential SF

1) *Differential Modulation*: Define a 2×2 matrix formed from the p th element of vectors $\mathbf{x}_i(2n+k)$, $i = 1, 2$, and $k = 0, 1$, as follows [cf. (1)]:

$$\mathbf{X}(n;p) \triangleq \begin{bmatrix} x_1(2n;p) & x_1(2n+1;p) \\ x_2(2n;p) & x_2(2n+1;p) \end{bmatrix}. \quad (5)$$

To initiate the differential SF modulation, we set

$$\mathbf{X}(n;0) = \sqrt{E_s} \mathbf{I}_2 \quad (6)$$

where E_s denotes the total energy emitted from all transmit antennas per subcarrier. For $p \geq 1$, the differential SF modulation iterates as follows:

$$\mathbf{X}(n;p) = \mathbf{X}(n;p-1) \mathbf{S}(n;p), \quad p = 1, 2, \dots, P-1 \quad (7)$$

where $\mathbf{S}(n;p)$ is formed via the Alamouti coding [9]

$$\mathbf{S}(n;p) \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} s(2n;p) & -s^*(2n+1;p) \\ s(2n+1;p) & s^*(2n;p) \end{bmatrix}. \quad (8)$$

Remark 1: Notice that the above SF performs differential modulation within one code matrix $\mathcal{X}(n)$, whereas the ST in [5] involves two code matrices. In particular, the latter utilizes the following iteration:

$$\mathcal{X}(n; p) = \mathcal{X}(n-1; p)\mathcal{S}(n; p), \quad n = 1, 2, \dots \quad (9)$$

which is initialized by setting $\mathcal{X}(0; p) = \sqrt{E_s}\mathbf{I}_2$. While this may seem like a minor change, it leads to quite different performance in time- and frequency-selective channels (see Section IV).

2) *Differential Demodulation and Decoding:* While multi-symbol detection is possible, we consider a standard “two-symbol” differential demodulation due to its simplicity. For joint demodulation and decoding, we need to collect signals received on all subcarriers assigned to one LCD codeword. To this end, let

$$\mathbf{y}_m(2n+k) \triangleq [y(2n+k; m), y(2n+k; m+M), \dots, y(2n+k; m+LM)]_{(L+1) \times 1}^T, \quad m=0, 1, \dots, M-1 \quad (10)$$

and let $\mathbf{x}_{i,m}(2n+k)$ be formed in a similar manner from $\mathbf{x}_i(2n+k)$. From (2), we have

$$\mathbf{y}_m(2n+k) = \sum_{i=1}^2 \mathbf{X}_{i,m}(2n+k)\mathbf{h}_{i,m} + \mathbf{w}_m(2n+k) \quad (11)$$

where $\mathbf{X}_{i,m}(2n+k) \triangleq \text{diag}\{\mathbf{x}_{i,m}(2n+k)\}$, $\mathbf{h}_{i,m} \triangleq [H_i(m), H_i(m+M), \dots, H_i(m+LM)]_{(L+1) \times 1}^T$, and $\mathbf{w}_m(2n+k)$ contains the noise samples on the $L+1$ subcarriers during the $(2n+k)$ th time slot. Next, let

$$\mathbf{y}_m(n) \triangleq [\mathbf{y}_m^T(2n), \mathbf{y}_m^T(2n+1)]_{2(L+1) \times 1}^T \quad (12)$$

and $\mathbf{w}_m(n)$ is similarly formed from $\mathbf{w}_m(2n+k)$. It follows from (11) that

$$\mathbf{y}_m(n) = \mathcal{X}_m^T(n)\mathbf{h}_m + \mathbf{w}_m(n) \quad (13)$$

where

$$\mathcal{X}_m(n) \triangleq \begin{bmatrix} \mathbf{X}_{1,m}(2n) & \mathbf{X}_{1,m}(2n+1) \\ \mathbf{X}_{2,m}(2n) & \mathbf{X}_{2,m}(2n+1) \end{bmatrix} \quad (14)$$

$$\mathbf{h}_m \triangleq [\mathbf{h}_{m,1}^T, \mathbf{h}_{m,2}^T]^T. \quad (15)$$

Note that the $(m-1)$ th and m th groups of subcarriers are next to each other in the frequency domain, which is due to the uniform interleaving (4). As such, we have [cf. (7)]

$$\mathcal{X}_m(n) = \mathcal{X}_{m-1}(n)\mathcal{S}_m(n) \quad (16)$$

where

$$\mathcal{S}_m(n) \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{S}_m(2n) & -\mathbf{S}_m^*(2n+1) \\ \mathbf{S}_m(2n+1) & \mathbf{S}_m^*(2n) \end{bmatrix} \quad (17)$$

with $\mathbf{S}_m(2n+k) \triangleq \text{diag}\{\mathbf{s}_m(2n+k)\}$, $k = 0, 1$. While the differential ST [5] assumes the channel remains (approximately) static over adjacent time slots, we assume that the difference of

the frequency response over adjacent subcarriers is negligible (also see *Remark 2* below)

$$\mathbf{h}_{m-1} \approx \mathbf{h}_m. \quad (18)$$

Then, substituting (17) into (13) yields

$$\begin{aligned} \mathbf{y}_m(n) &= \mathcal{S}_m^T(n)\mathcal{X}_{m-1}^T(n)\mathbf{h}_m + \mathbf{w}_m(n) \\ &= \mathcal{S}_m^T(n)\mathbf{y}_{m-1}(n) + \mathbf{v}_m(n) \end{aligned} \quad (19)$$

where $\mathbf{v}_m(n) \triangleq \mathbf{w}_m(n) - \mathcal{S}_m^T(n)\mathbf{w}_{m-1}(n)$. Since $\mathcal{S}_m^T(n)$ is unitary by construction, $\mathbf{v}_m(n)$ is complex Gaussian with zero-mean and covariance matrix $N_0\mathbf{I}_{2(L+1)}$. A least-squares (LS)-based decoding reduces to

$$\begin{aligned} &\{\hat{\mathbf{s}}_m(2n), \hat{\mathbf{s}}_m(2n+1)\} \\ &= \arg \min_{\mathbf{s}_m(2n), \mathbf{s}_m(2n+1) \in \mathcal{S}} \|\mathbf{y}_m(n) - \mathcal{S}_m^T(n)\mathbf{y}_{m-1}(n)\|^2 \end{aligned} \quad (20)$$

where \mathcal{S} denotes the codebook.

The above decoding can be further simplified by exploiting the unitary structure of $\mathcal{S}_m(n)$. Specifically, let $\bar{\mathbf{y}}_m(n) \triangleq [\mathbf{y}_m^T(2n), \mathbf{y}_m^H(2n+1)]^T$, $\mathbf{s}_m(n) \triangleq [\mathbf{s}_m^T(2n), \mathbf{s}_m^T(2n+1)]^T$, and

$$\bar{\mathbf{y}}_{m-1}(n) \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{Y}_{m-1}(2n-2) & \mathbf{Y}_{m-1}(2n-1) \\ \mathbf{Y}_{m-1}^*(2n-1) & -\mathbf{Y}_{m-1}^*(2n-2) \end{bmatrix}$$

where $\mathbf{Y}_{m-1}(2n+k) \triangleq \text{diag}\{\mathbf{y}_{m-1}(2n+k)\}$, $k = -1, -2$. Let $\bar{\mathbf{v}}_m(n)$ be formed from the noise samples as $\bar{\mathbf{y}}_m(n)$. It is easy to show that (19) is equivalent to

$$\bar{\mathbf{y}}_m(n) = \bar{\mathbf{Y}}_{m-1}(n)\mathbf{s}_m(n) + \bar{\mathbf{v}}_m(n). \quad (21)$$

Observe that $\bar{\mathbf{Y}}_{m-1}(n)$ has orthogonal columns/rows. Let $\bar{\boldsymbol{\Omega}}_{m-1}(n) \triangleq \mathbf{Y}_{m-1}^*(2n-2)\mathbf{Y}_{m-1}(2n-2) + \mathbf{Y}_{m-1}^*(2n-1)\mathbf{Y}_{m-1}(2n-1)$. It is ready to verify that

$$\bar{\mathbf{Z}}_{m-1}(n) \triangleq \bar{\mathbf{Y}}_{m-1}(n) \left[\mathbf{I}_2 \otimes \bar{\boldsymbol{\Omega}}_{m-1}^{-\frac{1}{2}}(n) \right] \quad (22)$$

is unitary. As such

$$\begin{aligned} \mathbf{z}_m(n) &\triangleq \bar{\mathbf{Z}}_{m-1}^H(n)\bar{\mathbf{y}}_m(n) \\ &= \left[\mathbf{I}_2 \otimes \bar{\boldsymbol{\Omega}}_{m-1}^{\frac{1}{2}}(n) \right] \mathbf{s}_m(n) + \mathbf{u}_m(n) \end{aligned} \quad (23)$$

where $\mathbf{u}_m(n) \triangleq \bar{\mathbf{Z}}_{m-1}^H(n)\bar{\mathbf{v}}_m(n)$. Let $\mathbf{z}_m(2n+k)$, $k = 0, 1$, be the k th length- $(L+1)$ subvector of $\mathbf{z}_m(n)$. It follows from (23) that the LS decoding reduces to

$$\begin{aligned} &\hat{\mathbf{s}}_m(2n+k) \\ &= \arg \min_{\mathbf{s}_m(2n+k) \in \mathcal{S}} \|\mathbf{z}_m(2n+k) - \bar{\boldsymbol{\Omega}}_{m-1}^{\frac{1}{2}}(n)\mathbf{s}_m(2n+k)\|^2 \\ &= \arg \max_{\mathbf{s}_m(2n+k) \in \mathcal{S}} \Re \left\{ \mathbf{z}_m^H(2n+k)\bar{\boldsymbol{\Omega}}_{m-1}^{\frac{1}{2}}(n)\mathbf{s}_m(2n+k) \right\} \end{aligned} \quad (24)$$

where the second equality is due to the fact that the coded symbols are drawn from a constant-energy PSK constellation.

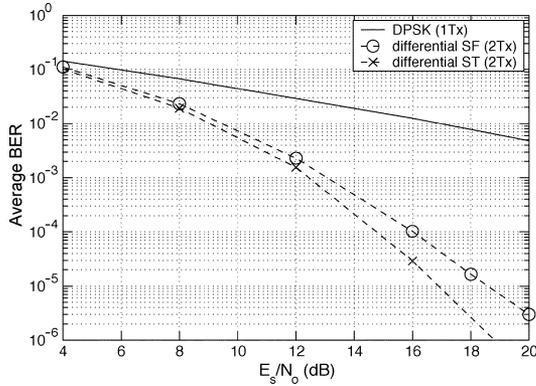


Fig. 2. Average BER versus SNR in frequency-selective block-fading channels.

Hence, the LS decoding is decoupled with respect to $\mathbf{s}_m(2n)$ and $\mathbf{s}_m(2n+1)$.

Remark 2: The approximation (18) may be inappropriate, especially in channels with severe frequency-selective fading that exhibits more rapid channel variation across adjacent subcarriers. This will cause some performance loss of the above SF scheme in such channels. As we shall see in Section IV, SF loses some resistance to frequency-selective fading to trade for more robustness against time-selective fading.

IV. NUMERICAL RESULTS

We consider an OFDM system with $P = 48$ subcarriers with two transmit antennas and one receive antenna. We compare the proposed differential SF scheme with the differential ST scheme [5], both using the LCD spectral encoding and interleaving as described in Section III-A. The performance metric is the average bit-error rate (BER).

We first consider a three-path ($L = 2$) Rayleigh fading channel for which the channel coefficients remain static within one frame but vary independently from frame to frame (viz., block fading). Fig. 2 depicts the average BER versus E_s/N_0 of the differential SF and ST. The conventional single-antenna differential PSK (DPSK), which offers no diversity, is also included as a baseline. The two multi-antenna schemes are seen to outperform the single-antenna DPSK due to spatio-spectral diversity and achieve a steeper BER-SNR slope. It is also seen that ST has a considerable edge over SF in this block fading channel, as the latter suffers from frequency-selective fading (see *Remark 2*).

We next consider a continuous-fading channel for which the channel coefficients vary continuously within a frame. Specifically, the channel coefficients are generated according to Channel Model A, as specified in HIPERLAN/2, and are time-varying with terminal speed v m/s and carrier frequency 5.2 GHz. Fig. 3 depicts the average BER (x -axis) of SF and ST versus v when $E_s/N_0 = 15$ dB. It is seen that for v up to 200 m/s, ST outperforms SF, and for higher terminal speed, ST degrades rapidly and is poorer than SF. While the terminal speed that can be handled by ST is fairly high, it should be recognized that the reason why ST still performs well at such a high terminal speed is due to the relatively short symbol duration of HIPERLAN/2, which is 2 μ s in this case. With such

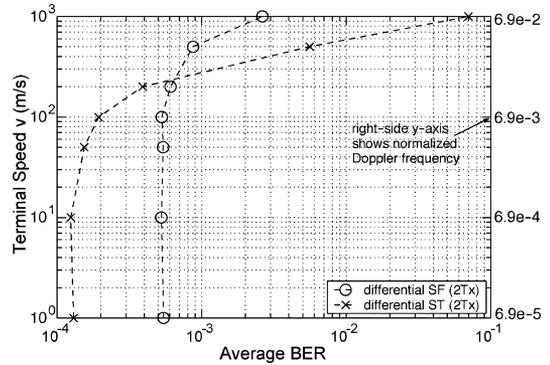


Fig. 3. Average BER versus terminal speed (normalized Doppler frequency) with $E_s/N_0 = 15$ dB in HIPERLAN/2 Model A channels.

a high symbol rate, the normalized Doppler frequency,¹ shown on the right side of Fig. 3, at $v = 200$ m/s is 6.9×10^{-3} , which is considered quite low, and the channel is effectively slow fading. For lower-rate applications when the fading speed becomes phenomenal, the advantage of SF becomes more prominent due to its ability to handle faster fading.

V. CONCLUSION

A new differential SF modulation scheme for coded OFDM with multiple transmit antennas in multipath channels is introduced. The differential SF scheme exhibits stronger resistance to time-selective fading, but less resistance to frequency-selective fading, than an earlier differential ST scheme. Our numerical results suggest that for broadband high-speed systems where the channel typically experiences slow fading, ST should be preferred to SF; meanwhile, for lower-rate applications when fast fading is prevalent, SF has an edge over ST.

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¹The normalized Doppler frequency is defined as $f_D T_s$, where f_D is Doppler frequency and T_s the symbol duration.