Performance of Differential Modulation with Wireless Relays in Rayleigh Fading Channels

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***Abstract*—We propose a new amplify-and-forward scheme amenable to differential modulation for cooperative systems with wireless relays. We derive closed-form expressions of the probability density function (PDF) of the signal-to-noise ratio (SNR) and average bit-error rate (BER) for the proposed scheme, and the analytical results are confirmed with numerical simulations. An asymptotic analysis reveals that the proposed cooperative scheme with one relay offers a diversity order approaching two in Rayleigh fading channels as the average SNR increases.**

***Index Terms*—Wireless relays, cooperative diversity, differential modulation, performance analysis.**

I. INTRODUCTION

DUE to the broadcasting nature, wireless transmission from one node to another can be heard by other nodes in the neighborhood. One or more such neighbor nodes may act as a *relay* by forwarding the received signal to the destination, which creates a unique *cooperative diversity*. Cooperative diversity techniques have received significant interest recently (e.g., [1]– [6] and references therein).

Most of the above studies, however, focus on coherent modulation, assuming that the channels can be reliably estimated at the relay and destination nodes. Meanwhile, channel estimation for the multiple (i.e., source-destination, source-relay(s), and relay(s)-destination) wireless links is costly and challenging in a fading environment. In this work, we examine differential modulation which obviates channel estimation for wireless relay systems. We propose a new amplify-and-forward relay scheme that, unlike the one in [3], does not need the instantaneous channel states and, thus, is more suitable for differential modulation. The proposed scheme is analyzed and shown to outperform the standard non-cooperative differential PSK without relays.

II. SYSTEM MODEL

Consider a scenario depicted in Fig. 1, where a sequence of symbols are to be transmitted from the *source* node S to the *destination* node D. Suppose there is another *relay* node R that can hear S and transmit to D. To avoid interference, S and R use orthogonal channels for transmission, either by time-, frequency-, or code-division multiplexing [3]. For ease of presentation, we assume time-division multiplexing for which the transmission is divided into *two* distinct phases. During *phase-I* transmission, S transmits, while R and D listen. During *phase-II* transmission, S is silent, while R amplifies and forwards its received signal to D. Although the setup is not different from those considered in earlier work (e.g., [3]), we propose a new amplify-and-forward scheme implemented at R that is amenable to differential modulation in fading channels.

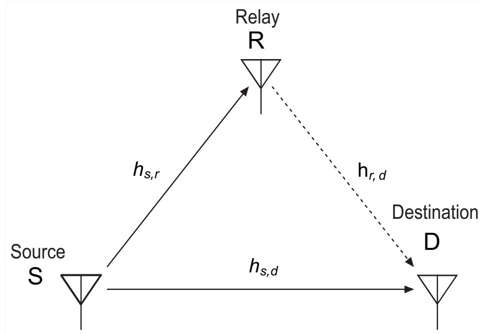

Fig. 1. A cooperative wireless relay system.

III. PROPOSED SCHEME

A. Two-Phase Transmission

During *phase-I*, the information bits $d(n) \in \{\pm 1\}$ at S are differentially encoded: $s(n) = s(n-1)d(n)$, $n = 1, 2, \cdots, N$, where $s(0) = 1$ and N is the number of bits within the frame. The received baseband signals at R and D, respectively, are

$$x_r(n) = h_{s,r}s(n) + w_r(n), \quad n = 0, 1, \cdots, N, \tag{1}$$

$$x_d(n) = h_{s,d}s(n) + w_d(n), \quad n = 0, 1, \cdots, N, \tag{2}$$

where $h_{s,r}$ and $h_{s,d}$ are the channel coefficients, and $w_r(n)$ and $w_d(n)$ are complex white Gaussian noise with zero-mean and variance N_0. The dependence on time of the time-varying channels are not shown to simplify notation. For differential demodulation, we use the standard assumption that the channels remain approximately unchanged within two symbol periods. We assume the channels are Rayleigh fading, i.e., $h_{s,r} \sim \mathcal{CN}(0, \sigma_{s,r}^2)$ and $h_{s,d} \sim \mathcal{CN}(0, \sigma_{s,d}^2)$, where $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable with mean μ and variance σ^2. The *instantaneous* SNR of the two links are $\gamma_{s,r} = |h_{s,r}|^2/N_0$ and $\gamma_{s,d} = |h_{s,d}|^2/N_0$, respectively, and the *average* SNR are $\bar{\gamma}_{s,r} = \sigma_{s,r}^2/N_0$ and $\bar{\gamma}_{s,d} = \sigma_{s,d}^2/N_0$, respectively.

For *phase-II* transmission, the signal at R is scaled/amplified as:

$$x'_r(n) \triangleq \frac{x_r(n)}{(\mathrm{var}\{x_r(n)\})^{1/2}} = \frac{x_r(n)}{(N_0 + \sigma_{s,r}^2)^{1/2}}. \tag{3}$$

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Clearly, $x'_r(n)$ has unit average energy. Note that our proposed scheme (3) differs from the one in [3] for *coherent* modulation:

$$x'_r(n) = x_r(n)(N_0 + |h_{s,r}|^2)^{-1/2}. \quad (4)$$

Specifically, (4) requires the magnitude of the *instantaneous* channel $h_{s,r}$, which is difficult to obtain in time-varying fading channels. Meanwhile, $\mathrm{var}\{x_r(n)\}$ in (3) can be conveniently estimated by time-averaging over a frame of received signals. Hence, our scheme is more suitable for *differential* modulation.

The signals received at D during phase II are

$$y_d(n) = h_{r,d}x'_r(n) + u_d(n), \quad n = 0, 1, \cdots, N, \quad (5)$$

where $h_{r,d} \sim \mathcal{CN}(0, \sigma^2_{r,d})$ and $u_d(n) \sim \mathcal{CN}(0, N_0)$. The instantaneous and average SNR are $\gamma_{r,d} = |h_{r,d}|^2/N_0$ and $\bar{\gamma}_{r,d} = \sigma^2_{r,d}/N_0$, respectively.

B. Differential Demodulation

Substituting (1) and (3) into (5) yields

$$y_d(n) = \tilde{h}_{s,d}s(n) + \tilde{w}_d(n), \quad n = 0, 1, \cdots, N, \quad (6)$$

where $\tilde{h}_{s,d}$ is the *effective channel* between S and D through R and $\tilde{w}_d(n)$ is the *effective channel noise*:

$$\tilde{h}_{s,d} \triangleq h_{s,r}h_{r,d}(N_0 + \sigma^2_{s,r})^{-1/2},$$
$$\tilde{w}_d(n) \triangleq h_{r,d}(N_0 + \sigma^2_{s,r})^{-1/2}w_r(n) + u_d(n).$$

Differential demodulation using *only* (2) or (6) is standard. Of more interest is to use both to seek additional diversity. It is seen from (2) and (6) that $x_d(n)$ and $y_d(n)$ are independent Gaussian random variables, conditioned on the channels and $s(n)$. Using the multichannel communication result of [7, Sect. 12.1], we can show that differential demodulation based on $x_d(n)$ and $y_d(n)$ amounts to the following combining (also see [8])

$$z(n) = x^*_d(n-1)x_d(n) + \frac{(N_0 + \sigma^2_{s,r})y^*_d(n-1)y_d(n)}{N_0 + \sigma^2_{s,r} + \sigma^2_{r,d}}, \quad (7)$$

where $(\cdot)^*$ denotes complex conjugation. Finally, the information bits are detected as follows: $\hat{d}(n) = \mathrm{sign}(\Re\{z(n)\})$.

IV. Analysis

We first consider the *relay link* (i.e., S-R-D link), and then include the *direct link* (i.e, S-D link) and examine the performance of combining both links.

A. Relay Link Only

The equivalent instantaneous SNR of the relay link is given by (see (6))

$$\gamma_{eq} = \frac{\gamma_{s,r}\gamma_{r,d}}{\bar{\gamma}_{s,r} + 1 + \gamma_{r,d}}. \quad (8)$$

Note the above expression is different from the instantaneous SNR in [9] that uses (4) for amplification. The PDF of γ_{eq} is derived in the Appendix:

$$\begin{aligned} p(\gamma_{eq}) =& 2\frac{\bar{\gamma}_{s,r}+1}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}}\exp\left(-\frac{\gamma_{eq}}{\bar{\gamma}_{s,r}}\right)K_0(\beta\sqrt{\gamma_{eq}}) + \frac{2}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}} \\ &\times\sqrt{\frac{\gamma_{eq}(1+\bar{\gamma}_{s,r})\bar{\gamma}_{r,d}}{\bar{\gamma}_{s,r}}}\exp\left(-\frac{\gamma_{eq}}{\bar{\gamma}_{s,r}}\right)K_1(\beta\sqrt{\gamma_{eq}}), \end{aligned} \quad (9)$$

where $\beta = 2\sqrt{\frac{1+\bar{\gamma}_{s,r}}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}}}$, $K_0(\cdot)$ denotes the zeroth-order modified Bessel function of the second kind, and $K_1(\cdot)$ denotes the first order modified Bessel function of the second kind.

The BER for the *relay link* is $P_{e1} = \int_0^\infty \frac{1}{2}e^{-\gamma_{eq}}p(\gamma_{eq})d\gamma_{eq}$. Let $\sqrt{\gamma_{eq}} = u$ and $\alpha = 1/\bar{\gamma}_{s,r} + 1$, we have

$$\begin{aligned} P_{e1} =& 2\frac{\bar{\gamma}_{s,r}+1}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}}\int_0^\infty ue^{-\alpha u^2}K_0(\beta u)du + \frac{2}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}} \\ &\times\sqrt{\frac{(1+\bar{\gamma}_{s,r})\bar{\gamma}_{r,d}}{\bar{\gamma}_{s,r}}}\int_0^\infty u^2e^{-\alpha u^2}K_1(\beta u)du \\ =& 0.5\bar{\gamma}_{r,d}^{-0.5}e^{0.5/\bar{\gamma}_{r,d}}W_{-0.5,0}(1/\bar{\gamma}_{r,d}) \\ &+ 0.5(1+\bar{\gamma}_{s,r})^{-1}e^{0.5/\bar{\gamma}_{r,d}}W_{-1,0.5}(1/\bar{\gamma}_{r,d}), \end{aligned} \quad (10)$$

where $W_{\lambda,\mu}(\cdot)$ denotes the Whittaker function [10] and we have used [10, Eqn. (6.631.3)].

B. Relay and Direct Links Combined

Note that γ_{eq} (8) is independent of $\gamma_{s,d}$, i.e., the instantaneous SNR of the direct (S-D) Rayleigh link, whose PDF is given by [7, Eqn. 14.3-5]

$$p(\gamma_{s,d}) = \frac{1}{\bar{\gamma}_{s,d}}e^{-\gamma_{s,d}/\bar{\gamma}_{s,d}}. \quad (11)$$

Hence, the instantaneous SNR γ at the output of the combiner (7) is the sum of the SNR of the direct and relay links:

$$\gamma = \gamma_{eq} + \gamma_{s,d}, \quad (12)$$

For binary differential PSK using multiple independent channels, the BER conditioned on γ is given by [7, Eqn. 12.1-13]

$$P_{e2}(\gamma) = \frac{1}{8}(4+\gamma)e^{-\gamma}. \quad (13)$$

Averaging the conditional BER with respect to the joint PDF of γ_{eq} and $\gamma_{s,d}$, we have (see [10, Eqn. (6.631.3)])

$$\begin{aligned} P_{e2} =& \int_0^\infty\int_0^\infty \frac{1}{8}(4+\gamma_{eq}+\gamma_{s,d})e^{-\gamma_{eq}-\gamma_{s,d}} \\ &\times p(\gamma_{eq})p(\gamma_{s,d})d\gamma_{eq}d\gamma_{s,d} \\ =& \exp\left(\frac{1}{2\bar{\gamma}_{r,d}}\right)\left[\frac{4+5\bar{\gamma}_{s,d}}{8(1+\bar{\gamma}_{s,d})^2\sqrt{\bar{\gamma}_{r,d}}}W_{-0.5,0}\left(\frac{1}{\bar{\gamma}_{r,d}}\right)\right. \\ &+ \frac{4+5\bar{\gamma}_{s,d}}{8(1+\bar{\gamma}_{s,d})^2(1+\bar{\gamma}_{s,r})}W_{-1,0.5}\left(\frac{1}{\bar{\gamma}_{r,d}}\right) \\ &+ \frac{\bar{\gamma}_{s,r}}{8(1+\bar{\gamma}_{s,d})(1+\bar{\gamma}_{s,r})\sqrt{\bar{\gamma}_{r,d}}}W_{-1.5,0}\left(\frac{1}{\bar{\gamma}_{r,d}}\right) \\ &\left.+ \frac{\bar{\gamma}_{s,r}}{4(1+\bar{\gamma}_{s,d})(1+\bar{\gamma}_{s,r})^2}W_{-2,0.5}\left(\frac{1}{\bar{\gamma}_{r,d}}\right)\right]. \end{aligned} \quad (14)$$

To gain some insight into (14), we examine the symmetric case with $\bar{\gamma}_{s,r} = \bar{\gamma}_{s,d} = \bar{\gamma}_{r,d} = \rho$ and let $\rho \to \infty$. For large ρ, (14) can be approximated as

$$\begin{aligned} P_{e2} \approx& \frac{1}{8}\rho^{-2}\left[5U(1,1,\rho^{-1}) + U(2,1,\rho^{-1})\right. \\ &\left.+ 5\rho^{-1}U(2,2,\rho^{-1}) + 2\rho^{-1}U(3,2,\rho^{-1})\right], \end{aligned} \quad (15)$$

where we used the result [11, Eqn. 13.1.33]:

$$W_{\lambda,\mu}(z) = e^{-z/2}z^{\mu+1/2}U(1/2+\mu-\lambda, 1+2\mu, z), \quad (16)$$

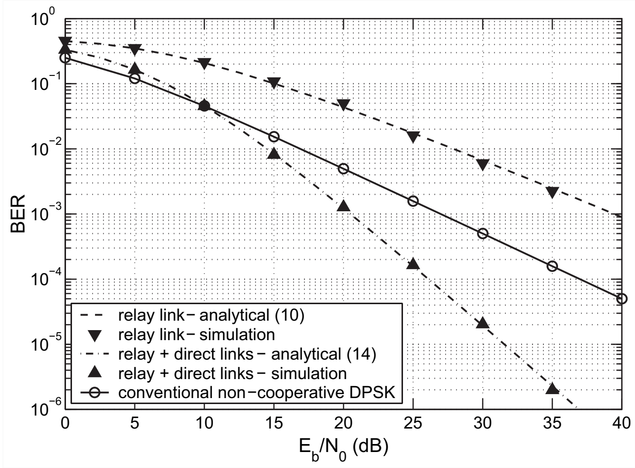

Fig. 2. Average BER in Rayleigh fading channels.

and $U(a, b, z)$ is the confluent hypergeometric function of the second kind. From [11, Eqns. 13.5.7 & 13.5.9], $U(1, 1, \rho^{-1})$ and $U(2, 1, \rho^{-1})$ behave like $\ln(\rho^{-1})$ for large ρ, while $U(2, 2, \rho^{-1})$ and $U(3, 2, \rho^{-1})$ behave like ρ for large ρ. Using these facts in (15) yields

$$P_{e2} \propto c\rho^{-2} \ln \rho, \quad \text{for large } \rho, \tag{17}$$

where c is a constant. Noting that $\lim_{\rho\to\infty} \rho^{-\epsilon} \ln \rho = 0$, $\forall\epsilon > 0$, we conclude that the diversity order of the system approaches 2 as $\rho \to \infty$.

V. Numerical Results

To verify the above analysis, we simulate the system shown in Fig. 1 in Rayleigh fading channels. We consider a symmetric scenario for which the average SNRs of all links are identical: $\bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = \bar{\gamma}_{s,d}$. We compare this cooperative system to a conventional non-cooperative system that involves direct transmission from S to D with binary differential PSK. For fair comparison, we set $\bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = \bar{\gamma}_{s,d} = 0.5E_b/N_0$, where E_b denotes the energy per bit, so that the sum of the transmitted energy from both S and R for the cooperative system is identical to the transmitted energy for the conventional system. Fig. 2 depicts (10), the average BER of the relay link only, and (14), the average BER of the relay and direct links combined, along with their simulation results, which are seen to agree with their analytical counterparts. Also shown there is the average BER of the conventional binary differential PSK. It is seen that the cooperative scheme outperforms the conventional non-cooperative scheme when $E_b/N_0 \geq 10$ dB, and achieves a diversity gain over the latter due to a steeper slope of the BER-SNR curve.

VI. Conclusions

We have introduced a new amplify-and-forward relay scheme for differential modulation/demodulation in cooperative wireless systems. We have obtained analytical expressions of the distribution of the instantaneous SNR and average BER for the proposed scheme, which has been shown to offer a full diversity advantage as the SNR goes to infinity.

Appendix

Let $X = \gamma_{s,r}\gamma_{r,d}$ and $Y = \bar{\gamma}_{s,r} + \gamma_{r,d} + 1$. The PDF of γ_{eq} is determined as follows (e.g., [12])

$$\begin{aligned} p(\gamma_{eq}) &= \int_{\bar{\gamma}_{s,r}+1}^{\infty} |y| p_{X,Y}(\gamma_{eq} y, y) dy \\ &= \int_{0}^{\infty} |\bar{\gamma}_{s,r} + 1 + \gamma_{r,d}| \\ &\quad \times p_{X,Y}(\gamma_{eq}(\bar{\gamma}_{s,r} + 1 + \gamma_{r,d}), (\bar{\gamma}_{s,r} + 1 + \gamma_{r,d})) d\gamma_{r,d}, \end{aligned}$$

where $p_{X,Y}(x, y) = p_{X|Y}(x|y)p_Y(y)$ denotes the joint PDF of X and Y. The marginal PDF of Y is given by

$$p_Y(y) = \bar{\gamma}_{r,d}^{-1} \exp(-(y - \bar{\gamma}_{s,r} - 1)/\bar{\gamma}_{r,d}).$$

The conditional PDF of $p_{X|Y}(x|y)$ is given by

$$p_{X|Y}(x|y) = \frac{1}{\bar{\gamma}_{s,r}|y - \bar{\gamma}_{s,r} - 1|} \exp\left(-\frac{x}{\bar{\gamma}_{s,r}(y - \bar{\gamma}_{s,r} - 1)}\right).$$

Substituting $p_Y(y)$ and $p_{X|Y}(x|y)$ back into the expression of $p(\gamma_{eq})$, followed by some manipulations, we have

$$\begin{aligned} p(\gamma_{eq}) =& \frac{\bar{\gamma}_{s,r} + 1}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}} \exp\left(-\frac{\gamma_{eq}}{\bar{\gamma}_{s,r}}\right) \int_0^{\infty} \frac{1}{\gamma_{r,d}} e^{(-a\gamma_{r,d} - b\gamma_{r,d}^{-1})} d\gamma_{r,d} \\ &+ \frac{1}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}} \exp\left(-\frac{\gamma_{eq}}{\bar{\gamma}_{s,r}}\right) \int_0^{\infty} e^{(-a\gamma_{r,d} - b\gamma_{r,d}^{-1})} d\gamma_{r,d}, \end{aligned}$$

where $a = 1/\bar{\gamma}_{r,d}$, $b = \gamma_{eq}(1 + \bar{\gamma}_{s,r})/\bar{\gamma}_{s,r}$. With the aid of [10, Eqn. 3.478.4], the PDF can be written as (9).

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