Capacity and GDoF Results for IC with Feedback & Receiver Cooperation

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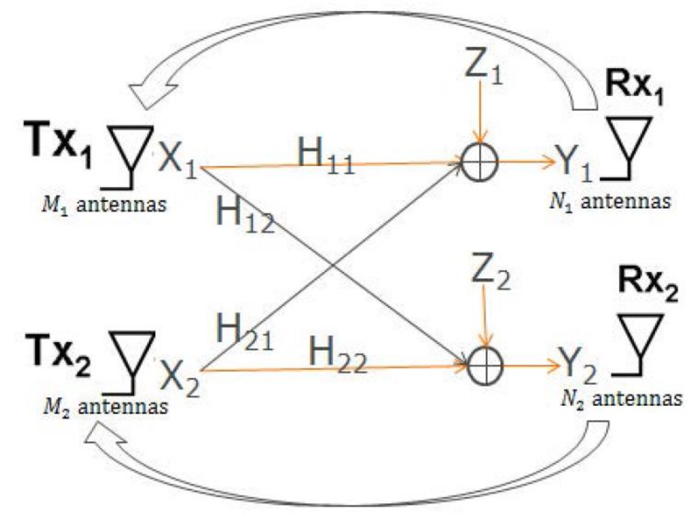
MIMO IC with Feedback*

System Model



$$Y_1[t] = \sqrt{\rho_{11}}H_{11}X_1[t] + \sqrt{\rho_{21}}H_{21}X_2[t] + Z_1[t],$$
$$Y_2[t] = \sqrt{\rho_{12}}H_{12}X_1[t] + \sqrt{\rho_{22}}H_{22}X_2[t] + Z_2[t].$$

Coding scheme: $X_i[t] = f_i(W_i, Y_i^{t-1}), i = 1, 2.$

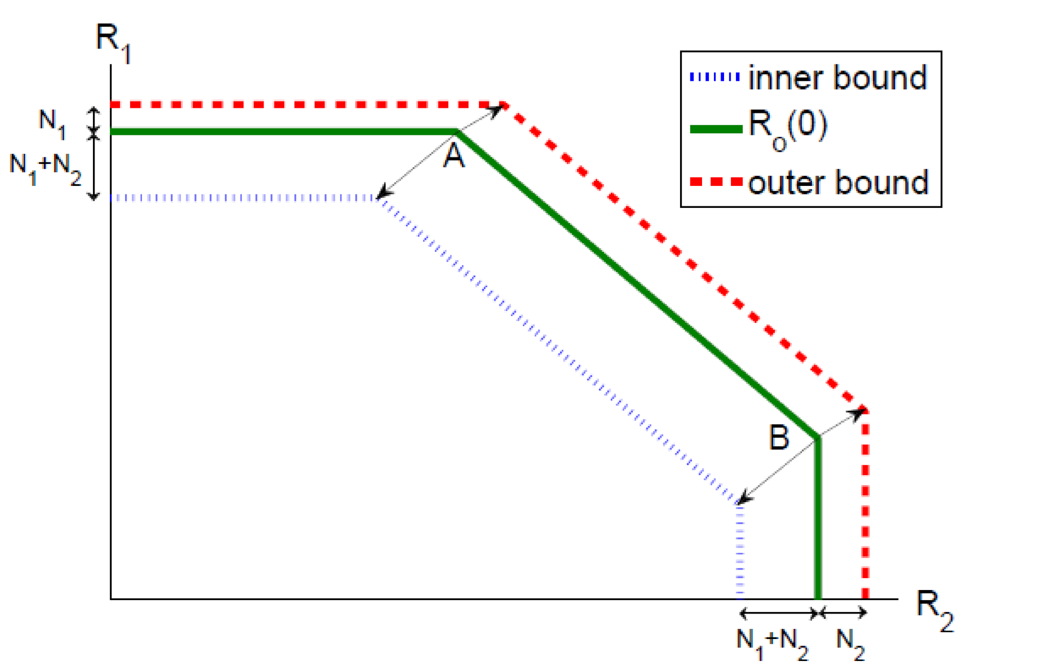
* M. Ashraphijuo, V. Aggarwal and X. Wang "On the Capacity Region and the Generalized Degrees of Freedom Region for the MIMO Interference Channel With Feedback" *IT Trans*, vol. 59, no. 12, pp. 8357-8376, Dec. 2013.

Approximate Capacity Region

Theorem: The capacity region for the two-user MIMO IC with pefect feedback $\mathbb{C}_{FB}$ is bounded from above and below as

$$\mathcal{R}_o(0) \ominus ([0, N_1+N_2] \times [0, N_1+N_2]) \subseteq \mathbb{C}_{FB} \subseteq \mathcal{R}_o(0) \oplus ([0, N_1] \times [0, N_2]),$$

where the inner and outer bounds are within $N_1 + N_2 + \max(N_1, N_2)$ bits and $\mathcal{R}_o(0)$ is given in [AAW'13, *Trans. IT*.].



Capacity Gain due to Feedback

Theorem: The capacity region for the two-user MIMO IC with pefect feedback $\mathbb{C}_{FB}$ is bounded from above and below as

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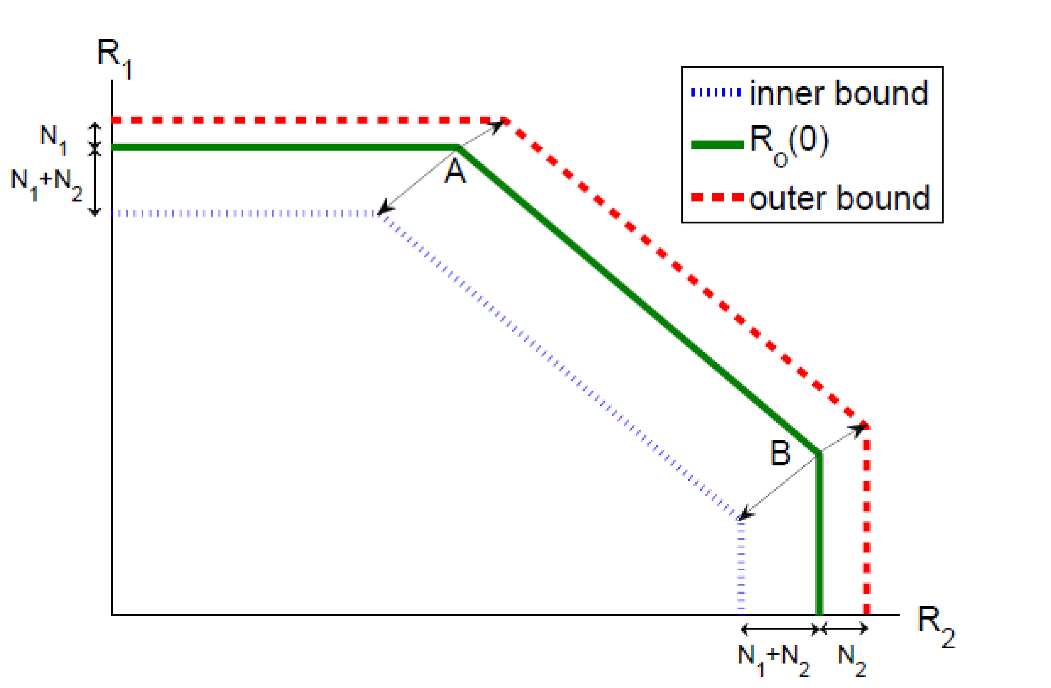


Generalized Degrees of Freedom

$GDoF = lim_{SNR\to\infty} \frac{R_{sym}}{\log SNR}$, $\alpha = \frac{\log INR}{\log SNR}$
M : # of Tx antennas, and N : # of Rx antennas.

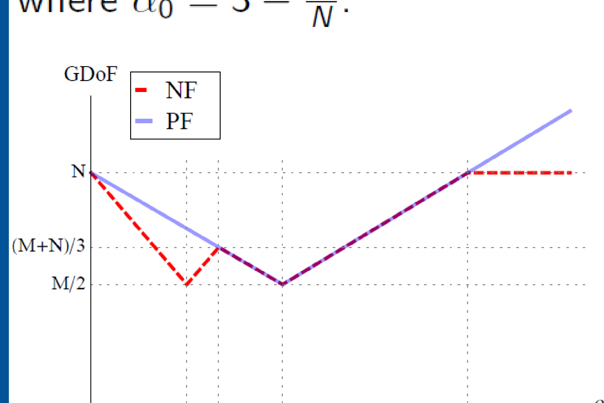
- Case $\frac{M}{2} < N \leq M$:

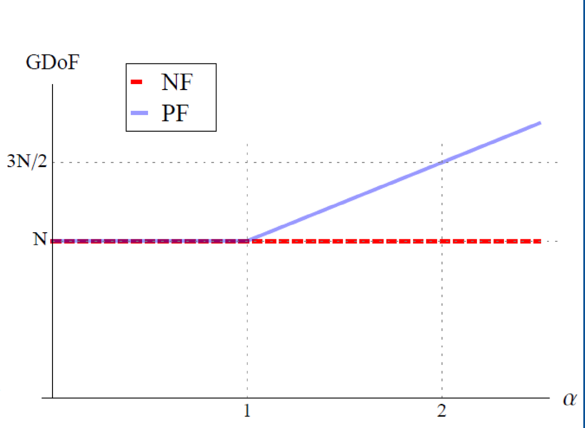
$GDoF = \max(N - (N - \frac{M}{2})\alpha, \frac{M-N}{2} + \frac{N\alpha}{2})$,

where $\alpha_0 = 3 - \frac{M}{N}$.

- Case $N \leq \frac{M}{2}$:

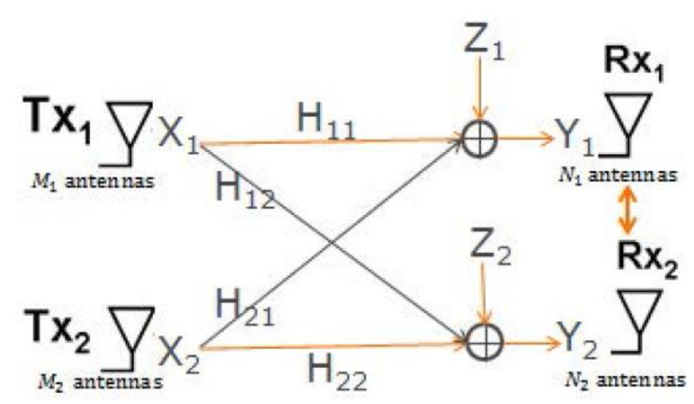
$GDoF = N \max(1, \frac{1+\alpha}{2})$.





MIMO IC with Limited Receiver Cooperation*

System Model



$$Y_1(t) = \sqrt{\rho_{11}}H_{11}X_1(t) + \sqrt{\rho_{21}}H_{21}X_2(t) + Z_1(t),$$
$$Y_2(t) = \sqrt{\rho_{12}}H_{12}X_1(t) + \sqrt{\rho_{22}}H_{22}X_2(t) + Z_2(t).$$

Decoding scheme: $\hat{m}_i = f_i(\Gamma_{ji}, Y_i(t))$,
Γ_{ji} : cooperation signal from Rx_j to Rx_i, $i \in \{1, 2\}$.

* M. Ashraphijuo, V. Aggarwal and X. Wang "On the Capacity and Degrees of Freedom Regions of Two-User MIMO Interference Channels With Limited Receiver Cooperation" *IT Trans*, vol. 60, no. 7, pp. 4170-4196, Jul. 2014.

Approximate Capacity Region

Theorem: The capacity region for the two-user MIMO IC with limited receiver cooperation $\mathbb{C}_{RC}$ is bounded from above and below as

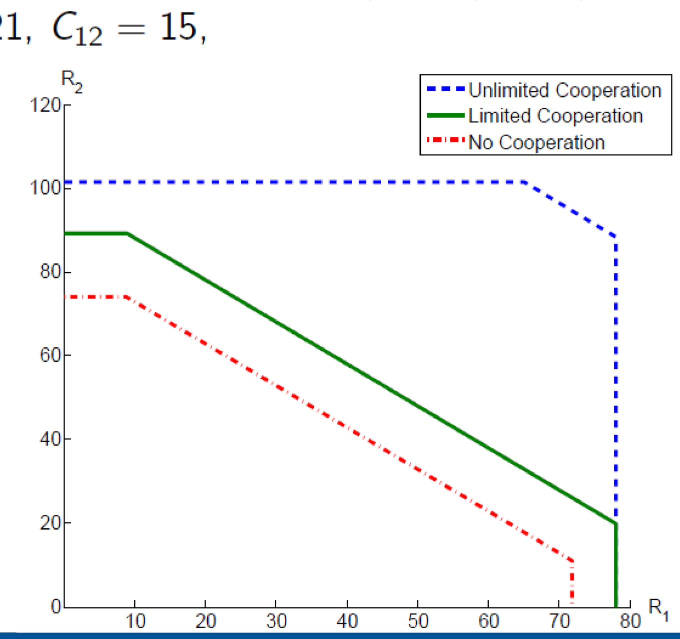
$$\mathcal{R}_o \ominus ([0, N_1+N_2] \times [0, N_1+N_2]) \subseteq \mathbb{C}_{RC} \subseteq \mathcal{R}_o,$$

where the inner and outer bounds are within $N_1 + N_2$ bits and $\mathcal{R}_o$ is given in [AAW'13].

$\mathcal{R}_o$ is a function of C_{coop}.

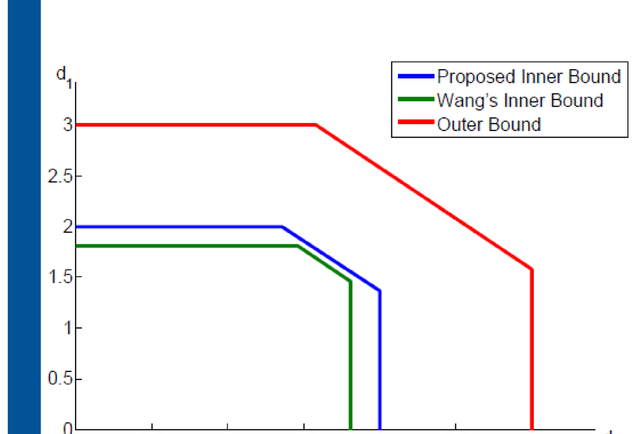
Capacity Gain due to Cooperation (Outer Bound)

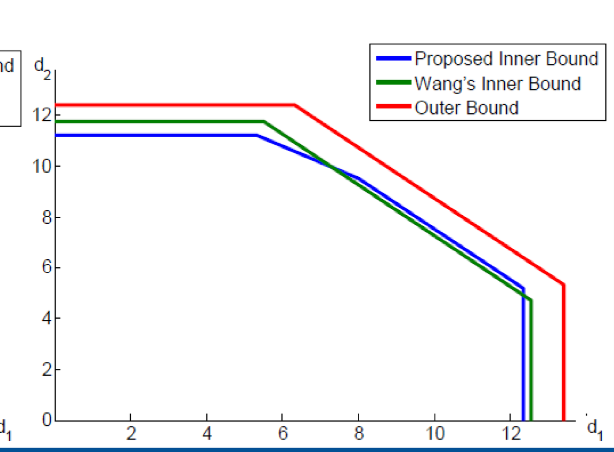
- $M_1 = N_2 = 3$, $M_2 = N_1 = 4$, $\rho_{11} = \rho_{22} = \rho_{12} = \rho_{21} = 10^8$, $C_{21} = 21$, $C_{12} = 15$,



Capacity Gain due to Cooperation (Inner Bound)

- Weak interference (left figure): $C_{21} = 1.1$, $C_{12} = 1.1$, $\text{SNR}_1 = 5$, $\text{SNR}_2 = 5$, $\text{INR}_1 = 2$ and $\text{INR}_2 = 2$.
- Strong interference (right figure): $C_{21} = 6$, $C_{12} = 11$, $\text{SNR}_1 = 1000$, $\text{SNR}_2 = 1500$, $\text{INR}_1 = 4000$ and $\text{INR}_2 = 10000$.

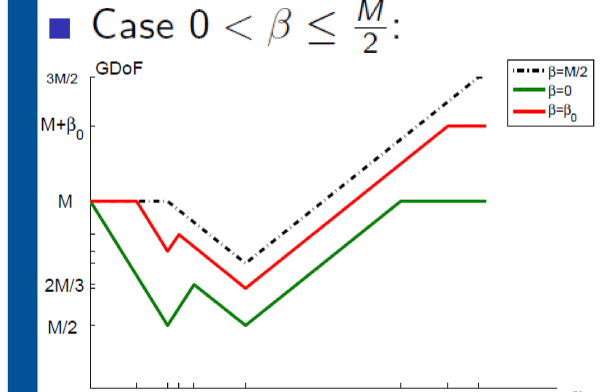




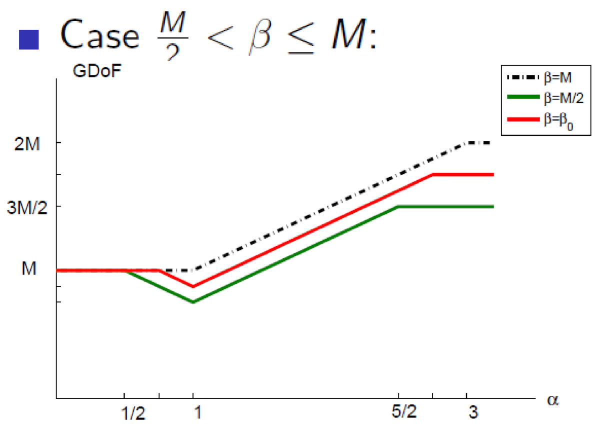
Generalized Degrees of Freedom

$GDoF = lim_{SNR\to\infty} \frac{R_{sym}}{\log SNR}$, $\alpha = \frac{\log INR}{\log SNR}$, $\beta = \frac{C_{coop}}{\log SNR}$,
M : # of Tx/Rx antennas.

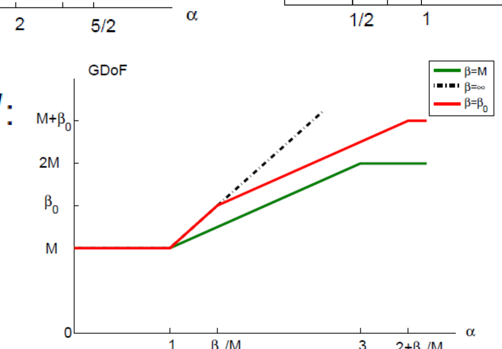
- Case $0 < \beta \leq \frac{M}{2}$:



- Case $\frac{M}{2} < \beta \leq M$:



- Case $\beta > M$:



K-User IC with Limited Feedback*

- Study this problem under two different interference channel models: the linear deterministic model, and the Gaussian model.
- Transmission strategy incorporates Han-Kobayashi message splitting, interference decoding, and decode-and-forward techniques.

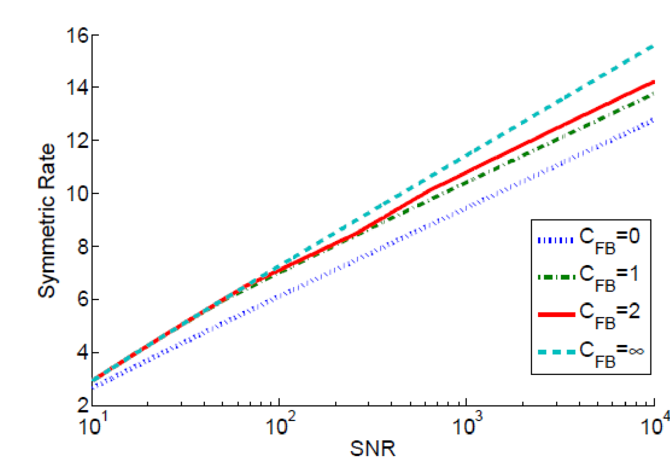
* M. Ashraphijuo, V. Aggarwal and X. Wang "On the Symmetric K-user Interference Channels with Limited Feedback" *Submitted to IT Trans*, Mar. 2014.

Previous Works

- For 2-user IC with limited feedback, [Vahid & Suh & Avestimehr'12, *Trans. IT*.] obtains approximate capacity region.
- For K-user IC with perfect feedback, [Mohajer & Tandon & Poor'13, *Trans. IT*.] found the approximate capacity region.
- For K-user IC with no feedback, [Jafar & Vishwanath'10, *Trans. IT*.] have capacity result.
- We obtain the approximate capacity region and DoF for K-user IC with limited feedback.

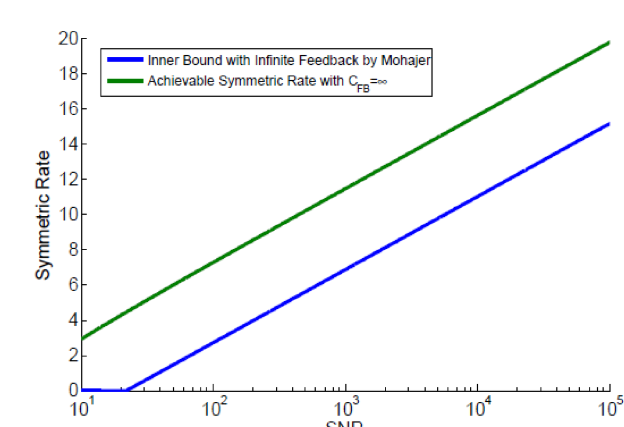
Capacity Gain due to Feedback

- Strong interference for $\alpha = \frac{\log INR}{\log SNR} = \frac{5}{2}$, and $K = 5$



Comparison

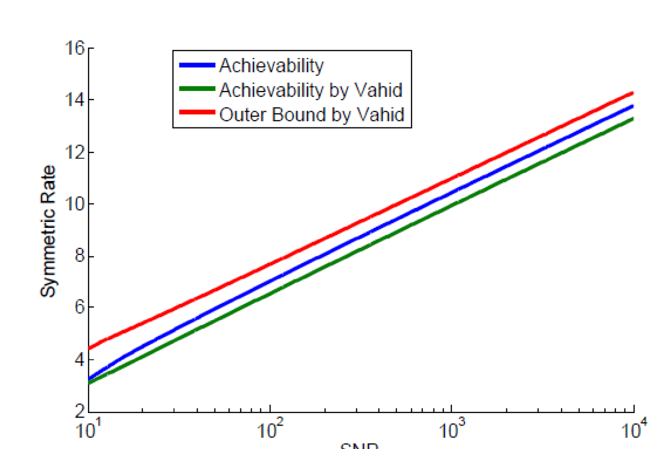
- Better achievability than [Mohajer & Tandon & Poor'13, *Trans. IT*.] in strong interference with infinite feedback.
- $K = 5$, and $\alpha = \frac{5}{2}$.



Similar improvements can be seen in other interference regimes, too.

Comparison

- Better achievability than [Vahid & Suh & Avestimehr'12, *Trans. IT*.] in the special case of 2-user in strong interference regime.
- $K = 2$, $C_{FB} = 1$, and $\alpha = \frac{5}{2}$.



- Our achievability scheme also fixes the result of [Vahid & Suh & Avestimehr'12, *Trans. IT*.] in the special case of 2-user in weak interference regime.

Ongoing Work

- Capacity Region of K-User MIMO Interference Channel with Arbitrary Number of Feedback Links
- Capacity Region of K-User MIMO Interference Channel with Arbitrary Topologies.
  - Non-Symmetric Models.
    - Non-Symmetric Topology.
    - Non-Symmetric Link Attenuations.
  - Arbitrary Number of Interference Links.