Single Sensor Blind Estimation of Time-Frequency Activity of a Mixture of Radio Signals via Block Tensor Decomposition

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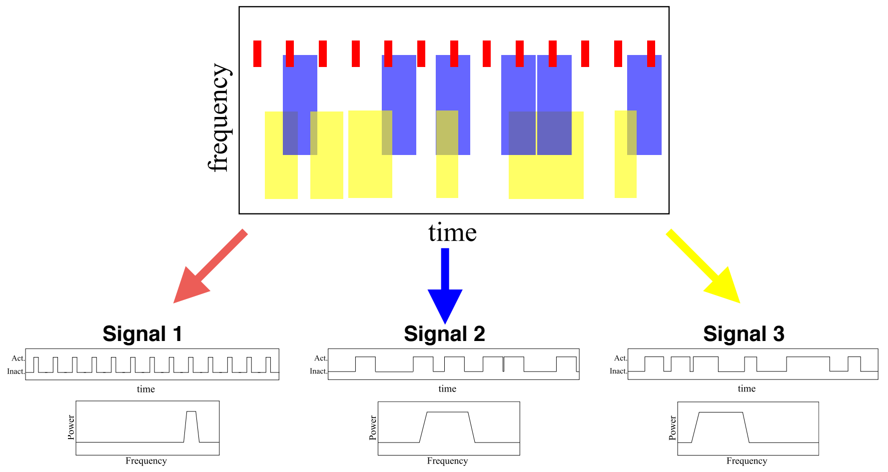
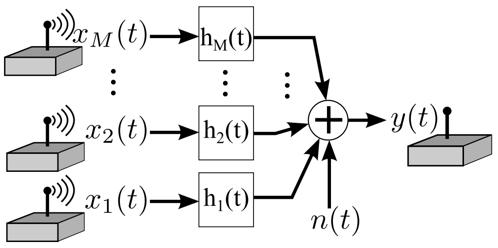
PROBLEM STATEMENT

- M transmitters are broadcasting in a bandwidth W
- Their transmissions are discontinuous in time
- Their transmissions are non-orthogonal in both time and frequency
- **Goal:** estimate each transmitter's signal parameters (activity in time, PSD, kurtosis) using a single sensor node



SYSTEM MODEL



M transmitters with modulations approximated by finite sum of linear modulations (AM, OFDM, etc.):

$$x_m(t) = \sum_{r=1}^{R_m} \sum_{k=-\infty}^{\infty} \underset{\substack{\uparrow \\ \text{i.i.d. symbols}}}{a_{m,r,k}} \underset{\substack{\uparrow \\ \text{pulse shape}}}{p_{m,r}}(t - kT_{m,r})$$

Transmissions are discontinuous in time (periods of time where transmitter is "off", $a_{m,r,k} = 0$ for some k).

Sensor receives:

$$y(t) = \sum_{m=1}^{M} h_m(t) * x_m(t) + n(t)$$

TRISPECTRUM SLICES

After preprocessing to find I_{cl} groups whose samples in-time have statistically similar signal contributions [1] we have a slice of the trispectrum for each group i=1,…, I_{cl}:

$$S_{04}(f,v,i) = \sum_{m=1}^{M} \underbrace{|H_m(f)|^2|H_m(v)|^2}_{\text{channel response m}} S_{04}^{x_m}(f,v)\underset{\substack{\uparrow \\ \text{on/off}}}{c_{im}} + \underbrace{S_{04}^{n}(f,v)}_{\text{trispectrum slice of noise = 0}}$$

Trispectrum slice of signal m:

$$S_{04}^{x_m}(f,v) = \sum_{r=1}^{R_m} \frac{S_{04}^{a_{mr}}(f,v)}{T_{mr}} |P_{mr}(f)|^2 |P_{mr}(v)|^2$$

Trispectrum slice of symbols of sub-signal r of signal m:

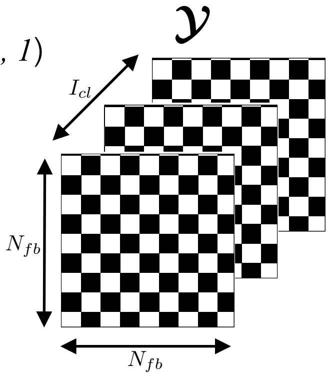
$$\frac{S_{04}^{a_{mr}}(f,v)}{T_{mr}} = \frac{\text{cum}_4(a_{mrk}, a_{mrk}, a_{mrk}^*, a_{mrk}^*)}{T_{mr}} = k_{mr}$$

BLOCK TENSOR STRUCTURE

After discretizing the trispectrum slices into N_{fb} frequency bins

$$y_{jni} = \int_{(j-1)\Delta f}^{j\Delta f} \int_{(n-1)\Delta f}^{n\Delta f} S_{04}^{y}(f,v,i)\, df\, dv = \sum_{m=1}^{M} c_{im} \sum_{r=1}^{R_m} k_{mr} f_{mrj} f_{mrn}$$

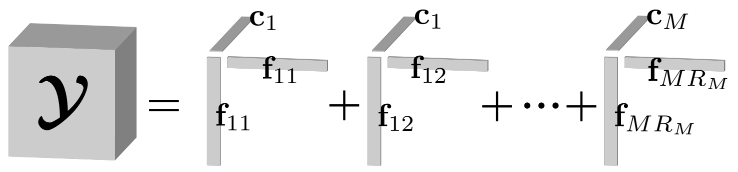
we see the tensor has a rank-(R_m, R_m, 1) block-partitioned tensor structure



$$\mathcal{Y} = \sum_{m=1}^{M} [\![\mathbf{k}_m; \mathbf{F}_m, \mathbf{F}_m, [\mathbf{c}_m \quad \dots \quad \mathbf{c}_m]]\!]$$

$$= \sum_{m=1}^{M} \sum_{r=1}^{R_m} k_{mr} \cdot \mathbf{f}_{mr} \circ \mathbf{f}_{mr} \circ \mathbf{c}_m$$

$$= \sum_{m=1}^{M} \mathbf{F}_m \cdot \text{diag}(\mathbf{k}_m) \cdot \mathbf{F}_m^T \circ \mathbf{c}_m$$



- Decomposing the tensor will allow us to estimate the desired parameters:

$$\mathbf{F}_m \rightarrow \text{PSD},\ \mathbf{c}_m \rightarrow \text{active/inactive sequence},\ \mathbf{k}_m \rightarrow \text{kurtosis}$$

FACTOR MATRIX ESTIMATION

- Block partition decomposition algorithms [2] assume prior knowledge of the partitioning.
- Our scenario: partitioning unknown, so we reformulate the tensor structure as

$$\mathcal{Y} = [\![\mathbf{k}; \mathbf{F}, \mathbf{F}, \mathbf{CV}]\!] = [\![\mathbf{F}, \mathbf{F}, \mathbf{CW}]\!]$$

where $\mathbf{W}$ is a sparse block-diagonal selection matrix.

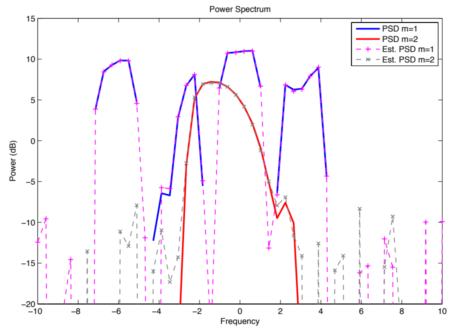
- Estimate factor matrices and partitions by solving:

$$\left[\hat{\mathbf{F}}, \hat{\mathbf{C}}, \hat{\mathbf{W}}\right] = \underset{\mathbf{F},\mathbf{C},\mathbf{W}}{\arg\min} \frac{1}{2} \|\mathcal{Y} - [\![\mathbf{F}, \mathbf{F}, \mathbf{CW}]\!]\|_2^2 + \lambda \|\mathbf{W}\|_1$$

- For each column, $\mathbf{w}_r$, fix $w_{mr} = 0$ if $m \neq \underset{m}{\max} |w_{mr}|$
- Redo minimization to find final estimates of $\mathbf{F}$, $\mathbf{C}$, and $\mathbf{k}$

SIMULATION

- We ran limited simulations at high SNR with an 8-tap random Rayleigh fading channel to test the operation of the algorithm.
- Algorithm successfully estimated signal parameters for appropriately selected L1 regularization coefficient for overlapping signals using Root Raised Cosine pulse shape (W=80Mhz).
- Tested Configurations:
  - M=2, R_m={1,1}, identical bandwidths, different f_c
  - M=4, R_m={1,1,1,1}, identical bandwidths, different f_c
  - M=2, R_m={3 narrowband ,2 wideband}, different f_c
- More simulations need to be run to fully quantify the algorithm performance.



Example PSD Estimate

FUTURE WORK

- Improve decomposition speed and scalability
- Selection of L1 regularization coefficient
- Extensive Monte Carlo simulations to quantify algorithm performance.

[1] G. Ivkovic, P. Spasojevic, and I.Seskar,"Mean shift based segmentation for time frequency analysis of packet based radio signals," in 2010 Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers, Nov 2010, pp. 1526–1530.

[2] L. De Lathauwer and D. Nion, "Decompositions of a Higher-Order Tensor in Block Terms—Part III: Alternating Least Squares Algorithms," SIAM. J. Matrix Anal. & Appl., vol. 30, no. 3, pp. 1067–1083, Jan. 2008.