Delay-Constrained Distortion Minimization for Energy Harvesting Transmission over a Fading Channel

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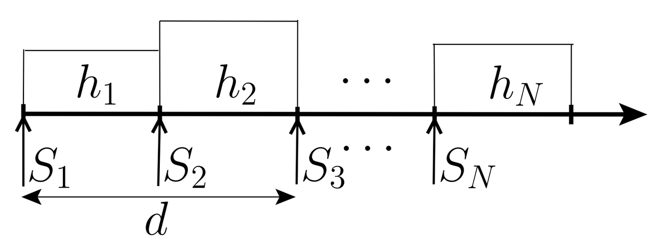
I. Motivation

- Wireless sensor networks:
  - Source and channel statistics vary over time.
  - Delay-sensitive terminals are power/energy limited.
    - $\Rightarrow$ Solution: Energy harvesting
- Challenge:
  - Management of limited energy to ensure minimal reconstruction distortion and delay at the fusion center.
  - Stochastic nature of source, channel, and energy arrivals.

II. System Model



- Time-varying source
  - $S_1, \ldots, S_N$ arrive sequentially in time.
  - Sources in different time slots have possibly different statistics.
    - $S_i \sim N(0, \sigma_i^2)$
- Time-varying channel
  - Channel power gain h_i varies from slot-to-slot, is constant in each time slot.
  - AWGN with unit variance.
- Delay
  - Source S_i needs to be delivered in at most d time slots.
- Energy harvested in time slot i is E_i.
- Energy is consumed only for data transmission.
- *Offline optimization:* σ_i^2, h_i, and E_i known non-causally at the transmitter.
- Goal: Minimize total distortion at the destination at the end of slot N.

III. Distortion Minimization

- Distortion (mean squared error) of S_i is D_i.
- p_j: Power allocated to slot j.
- Objective is to minimize $D \triangleq \sum_{i=1}^N D_i$.
- Constraints
  - Source causality constraint;
    $$\sum_{j=i}^N \frac{1}{2}\log\left(\frac{\sigma_j^2}{D_j}\right) \le \sum_{j=i}^N \frac{1}{2}\log(1+h_j p_j), \quad i=1,\ldots,N,$$
  - Delay constraint;
    $$\sum_{j=k}^i \frac{1}{2}\log\left(\frac{\sigma_j^2}{D_j}\right) \le \sum_{j=k}^{i+d-1} \frac{1}{2}\log(1+h_j p_j),$$
    $$i=k,\ldots,N-d, \quad k=1,\ldots,N-d.$$
- Source-channel separation is optimal.

IV. Distortion Minimization for Battery-Run System

- $E_i = 0$, $i > 1$, arbitrary d
- Let $r_i \triangleq \frac{1}{2}\log\left(\frac{\sigma_i^2}{D_i}\right)$ and $c_i \triangleq \frac{1}{2}\log(1+h_i p_i)$
- Optimization problem can be written in convex form

$$\underset{r_i, c_i}{\text{minimize}} \quad \sum_{i=1}^N \sigma_i^2 e^{-2r_i}$$

$$\text{subject to} \quad \sum_{i=1}^N \frac{e^{2c_i}-1}{h_i} \le E_1, \qquad \text{Energy causality}$$

$$\sum_{j=i}^N r_j \le \sum_{j=i}^N c_j, \quad i=1,\ldots,N, \qquad \text{Source causality}$$

$$\sum_{j=k}^i r_j \le \sum_{j=k}^{i+d-1} c_j, \quad i=k,\ldots,N-d, \quad k=1,\ldots,N-d, \qquad \text{Delay}$$

$$e^{-2r_i} \le 1 \quad \text{and} \quad 0 \le c_i, \quad i=1,\ldots,N,$$

Optimal distortion allocation:

The optimal distortion D_i^* satisfies

$$D_i^* = \begin{cases} \xi_i, & \text{if } \xi_i < \sigma_i^2, \\ \sigma_i^2, & \text{if } \xi_i \ge \sigma_i^2, \end{cases}$$

with

$$\xi_i = \begin{cases} \frac{1}{2}\sum_{j=1}^i \gamma_j + \frac{1}{2}\sum_{k=1}^i \sum_{j=i}^{N-d} \delta_{j,k}, & i=1,\ldots,N-d \\ \frac{1}{2}\sum_{j=1}^i \gamma_j, & i=N-d+1,\ldots,N, \end{cases}$$

where γ_i and $\delta_{i,j}$ are Lagrange multipliers corresponding to source causality and delay constraints, respectively.

- ξ_i can be interpreted as the *reverse water level* similar to the classical reverse waterfilling solution for parallel Gaussian sources.

Optimal power allocation:

The optimal power p_i^* satisfies

$$p_i^* = \left(\nu_i - \frac{1}{h_i}\right)^+, \quad \forall i,$$

where $\nu_i = \frac{\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k}}{2\lambda}$ and $\delta_{j,k} = 0$ for $j = 2-d, \ldots, 0, \forall k$.

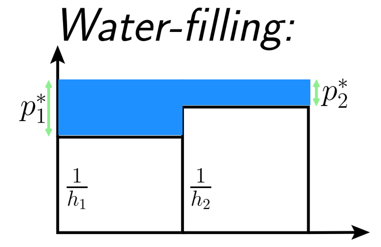
- ν_i can be interpreted as *water level* in the classical waterfilling solution.

Water level ν_i and reverse water level ξ_i depend on time slot i due to causality and delay constraints.

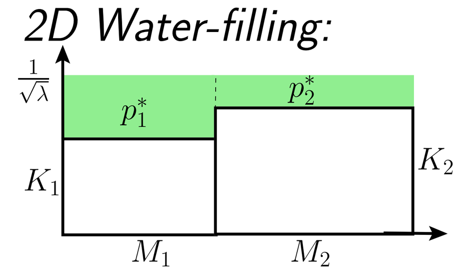
Strict delay constraint $d=1$

Solution: 2D Water-filling

Water-filling:



2D Water-filling:



- $M_i \triangleq \frac{\sigma_i}{\sqrt{h_i}}$, $K_i \triangleq \frac{1}{\sigma_i\sqrt{h_i}}$.
- p_i^* is given by the green area below the water level $\frac{1}{\sqrt{\lambda}}$.

V. Distortion Minimization with Energy Harvesting

- Energy causality constraint:

$$\sum_{j=1}^i p_j \le \sum_{j=1}^i E_j, \quad i=1,..,N.$$

Objective:

- Choose p_j, D_j, $j=1,\ldots,N$ to minimize total distortion D subject to source causality, delay, and energy causality constraints.

Optimal distortion & power allocation:

- The optimal distortion is the same as the battery-run case.
- The optimal power level p_i^* satisfies

$$p_i^* = \left(\pi_i - \frac{1}{h_i}\right)^+, \quad \forall i,$$

where $\pi_i = \frac{\sum_{j=1}^i \gamma_j + \sum_{k=1}^i \sum_{j=i-d+1}^{N-d} \delta_{j,k}}{2\sum_{j=i}^N \lambda_j}$, λ_i is Lagrange multiplier for the energy causality constraint.

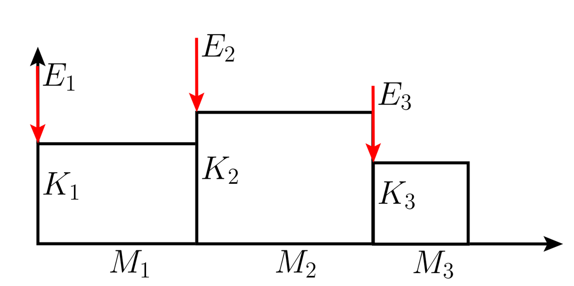
From complementary slackness conditions;

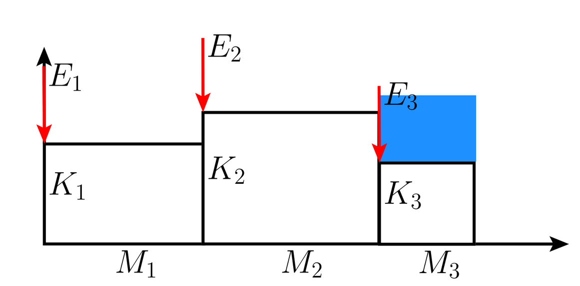
- $\gamma_i > 0 \Rightarrow$ total collected data until time slot i must be transmitted until time slot i.
- $\lambda_i > 0 \Rightarrow$ the battery must be depleted.

Strict delay constraint $d=1$

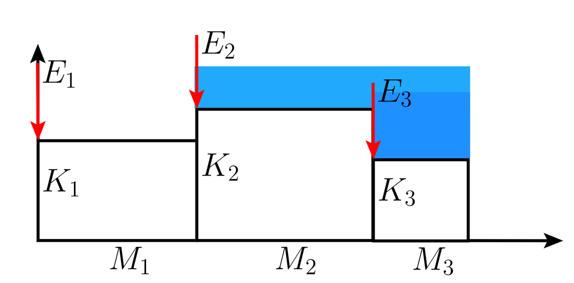
Solution: Directional 2D water-filling

Step 1:

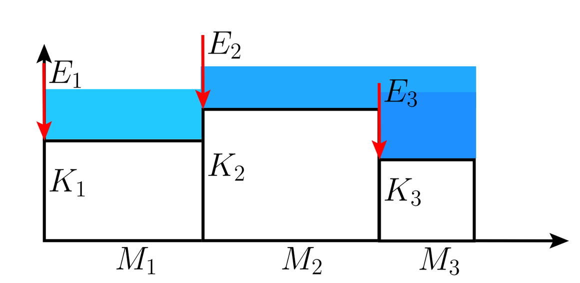




Step 2:



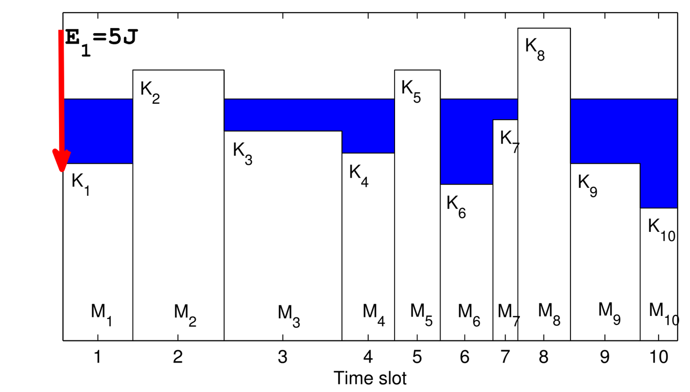
Last step:



- $M_i \triangleq \frac{\sigma_i}{\sqrt{h_i}}$, $K_i \triangleq \frac{1}{\sigma_i\sqrt{h_i}}$.
- Allocate energy packets to time slots starting from the last non-zero energy packet.
- The harvested energy E_i can only be allocated to time slots $j \ge i$.
- p_i^* is given by the colored area below the water level of time slot i in the last step.

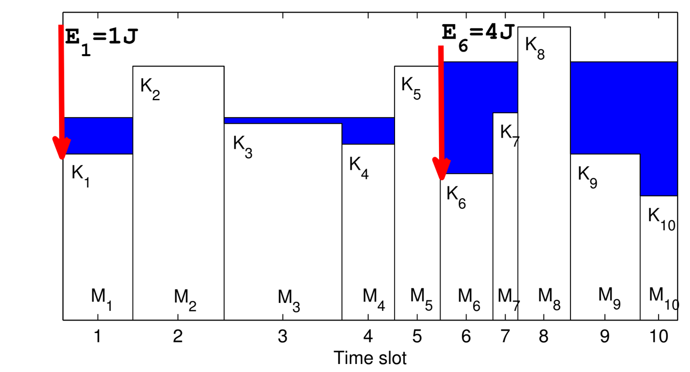
VI. Numerical Results

Battery-run systems, $d=1$



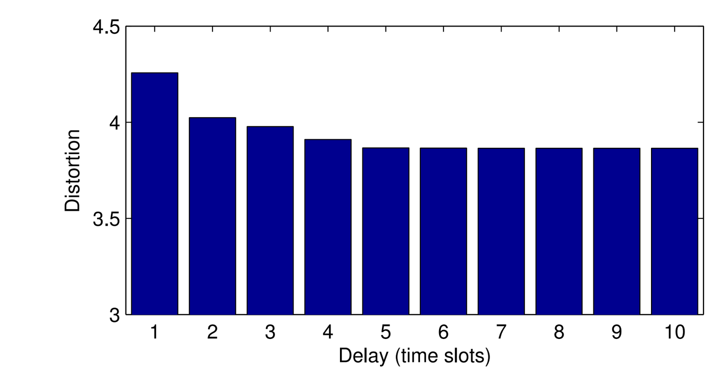
- $E_1 = 5$J, $N=10$, $d=1$
- The optimal distortion is $D = 4.32$.

Energy harvesting systems, $d=1$



- $E_1 = 1, E_6 = 4$ and $E_i = 0$J otherwise, $N=10$, $d=1$
- The optimal distortion is $D = 4.37$.

Battery-run system, arbitrary d



- $E_1 = 5$, $N = 10$
- The optimal distortion decreases monotonically for $d \le 4$ and remains constant after $d = 4$.

VII. Conclusions

- Studied distortion minimization for time varying source and channel with energy harvesting transmitter and under delay constraints.
- Provided convex optimization formulation.
- Identified optimal distortion and power allocation, and properties.
- Provided 2D water-filling (regular/directional) for strict delay $d=1$ case.

For more details, see: O. Orhan, D. Gunduz, and E. Erkip, "Delay-constrained distortion minimization for energy harvesting transmission over a fading channel," *IEEE International Symposium on Information Theory*, Istanbul, Turkey, Jul. 2013.