I. Motivation

- Wireless sensor networks:
  - Source and channel statistics vary over time.
  - Delay-sensitive terminals are power/energy limited.
- Solution: Energy harvesting

- Challenge:
  - Management of limited energy to ensure minimal reconstruction distortion and delay at the fusion center.
  - Stochastic nature of source, channel, and energy arrivals.

II. System Model

- Time-varying source:
  - \(S_1, \ldots, S_N\) arrive sequentially in time.
  - Sources in different time slots have possibly different statistics.
  - \(S_k \sim \mathcal{N}(\mu_k, \sigma_k^2)\)

- Time-varying channel:
  - Channel power gain \(h_k\) varies from slot-to-slot, is constant in each time slot.
  - AWGN with unit variance.
- Delay:
  - Source \(S_i\) needs to be delivered in at most \(d_i\) time slots.
- Energy harvested in time slot \(i\) is \(E_i\).
- Energy is consumed only for data transmission.
- Offline optimization: \(c_i, h_i\), and \(E_i\) known non-causally at the transmitter.
- Goal: Minimize total distortion at the destination at the end of slot \(N\).

III. Distortion Minimization

- Distortion (mean squared error) of \(S_i\) is \(D_i\).
- \(p_i\): Power allocated to slot \(i\).
- Objective is to minimize \(D = \sum_{i=1}^{N} D_i\).

- Constraints:
  - Source causality constraint:
    \[
    \sum_{j=1}^{i-1} \frac{h_j c_j}{p_j} \leq \sum_{j=1}^{i-1} \frac{h_j}{p_j} \log(1 + h_j p_j), \quad i = 1, \ldots, N.
    \]
  - Delay constraint:
    \[
    \sum_{j=1}^{i} \frac{h_j c_j}{p_j} \leq \sum_{j=1}^{i} \frac{h_j}{p_j} \log(1 + h_j p_j), \quad i = k, \ldots, N - d, \quad k = 1, \ldots, N - d.
    \]
  - Channel power gain \(h_i\) and source causality of \(S_i\) depend on time slot \(i\) due to causality and delay constraints.

IV. Distortion Minimization for Battery-Run System

- \(E_i = 0, i > 1\), arbitrary \(d\)
- Let \(r_i = \frac{1}{2} \log \left(\frac{1}{c_i} \right)\) and \(e_i = \frac{1}{2} \log(1 + h_i p_i)\)
- Optimization problem can be written in convex form

\[
\text{minimize}_{c_i, p_i} \sum_{i=1}^{N} \sigma_i^2 e_i^{2r_i}
\]

subject to
- \(\sum_{i=1}^{N} c_i = 1\), \(\forall i\), Energy causality
- \(\sum_{i=1}^{N} c_i \leq \sum_{j=1}^{N} e_j\), \(i = 1, \ldots, N\), Source causality
- \(e_i \leq 2r_i \leq 1\) and \(0 \leq e_i, i = 1, \ldots, N\), Delay constraint

Opportunistic allocation:
The optimal distortion \(D^*\) satisfies

\[
D^*_i = \begin{cases} \xi_i, & \text{if } \xi_i < \sigma_i^2, \\ \sigma_i^2, & \text{if } \xi_i \geq \sigma_i^2, \end{cases}
\]

with

\[
\xi_i = \frac{1}{2} \sum_{j=1}^{i-1} \gamma_j + \frac{1}{2} \sum_{j=1}^{i-1} \sum_{k=j+1}^{N} \delta_{ik}, \quad i = 1, \ldots, N - d
\]

\[
\delta_{ik} = \frac{1}{2} \sum_{j=1}^{i-1} \gamma_j, \quad i = N - d + 1, \ldots, N
\]

where \(\gamma_j\) and \(\delta_{ik}\) are Lagrange multipliers corresponding to source causality and delay constraints, respectively.

- \(\xi_i\) can be interpreted as the reverse water level similar to the classical reverse waterfilling solution for parallel Gaussian sources.

Optimal power allocation:
The optimal power \(p_i^*\) satisfies

\[
p_i^* = \left( \frac{\mu_i}{\lambda_i} \right)^{1}, \quad \forall i
\]

where \(\lambda_i = \sum_{j=1}^{i-1} \gamma_j + \sum_{j=1}^{i-1} \sum_{k=j+1}^{N} \delta_{ik}\) and \(\gamma_j = 0\) for \(j = 2, \ldots, N, \forall k\).

- \(\nu_i\) can be interpreted as water level in the classical waterfilling solution.

Water level \(p_i\) and reverse water level \(\xi_i\) depend on time slot \(i\) due to causality and delay constraints.

Strict delay constraint \(d = 1\):

\[
\text{Solution: } \nu_i = \frac{1}{2} \sum_{j=1}^{i-1} \gamma_j + \frac{1}{2} \sum_{j=1}^{i-1} \sum_{k=j+1}^{N} \delta_{ik}, \quad i = 1, \ldots, N - d
\]

\[
\text{Water-filling: } p_i\Rightarrow k_1, k_2, \ldots, k_N
\]

\[
\text{2D Water-filling: } M_1 \Leftrightarrow k_1, M_2 \Leftrightarrow k_2, \ldots, M_N \Leftrightarrow k_N
\]

- \(p_i^*\) is given by the colored area below the water level \(\xi_i\).

V. Distortion Minimization with Energy Harvesting

- Energy causality constraint:

\[
\sum_{i=1}^{N} p_i \leq \sum_{i=1}^{N} E_i, \quad i = 1, \ldots, N
\]

Objective:
- Choose \(p_i, D_i, i = 1, \ldots, N\) to minimize total distortion \(D\) subject to source causality, delay, and energy causality constraints.

Optimal distortion & power allocation:
- The optimal distortion is the same as the battery-run case.
- The optimal power level \(p_i^*\) satisfies

\[
p_i^* = \left( \frac{\mu_i}{\lambda_i} \right)^{1}, \quad \forall i
\]

where \(\nu_i = \sum_{j=1}^{i-1} \gamma_j + \sum_{j=1}^{i-1} \sum_{k=j+1}^{N} \delta_{ik}\)

- \(\lambda_i\) is Lagrange multiplier for the energy causality constraint.

From complementary slackness conditions:
- \(\gamma_i > 0\) if total collected data until time slot \(i\) must be transmitted until time slot \(i\).
- \(\lambda_i > 0\) if the battery must be depleted.

Strict delay constraint \(d = 1\):

\[
\text{Solution: Directional 2D water-filling}
\]

Step 1:

- \(p_i^*\) is given by the colored area below the water level.

Step 2:

- \(M_i \Leftrightarrow k_i\)

Step 3:

- \(M_i \Leftrightarrow k_i\)

VI. Numerical Results

- Battery-run systems, \(d = 1\):

- \(E_1 = 5\), \(N = 10\), \(d = 1\)
- The optimal distortion is \(D = 4.32\).

- Energy harvesting systems, \(d = 1\):

- \(E_1 = 1\), \(E_2 = 4\) and \(E_3 = 0\) otherwise, \(N = 10\), \(d = 1\)
- The optimal distortion is \(D = 4.37\).

- Battery-run system, arbitrary \(d\):

- \(E_1 = 5\), \(N = 10\)
- The optimal distortion decreases monotonically for \(d \leq 4\) and remains constant after \(d = 4\).

VII. Conclusions

- Studied distortion minimization for time varying source and channel with energy harvesting transmitter and under delay constraints.
- Provided convex optimization formulation.
- Identified optimal distortion and power allocation, and properties.
- Provided 2D water-filling (regular/directional) for strict delay \(d = 1\) case.