Study of Memory Effects in International Market Indices

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Abstract. Long term memory effects in stock market indices that represent internationally diversified stocks are analyzed in this paper and the results are compared with the S&P 500 index. The Hurst exponent and the Detrended fluctuation analysis (DFA) technique are the tools used for this analysis. The financial time-series data of these indices are tested with the Normalized Truncated Levy Flight to check whether the evolution of these indices is explained by the *TLF*.

Some features that seem to be specific for international indices are discovered and briefly discussed. In particular, a potential investor seems to be faced with new investment opportunities in emerging markets during and especially after a crisis.

Keywords: EAFE Index, International stock market indices, Detrended fluctuation analysis, R/S Analysis

1. Introduction

In recent years there has been a growing literature in financial economics that analyzes the major stock indices in developed countries, see for example [1-11] and the references therein. The statistical properties of the temporal series analyzing the evolution of the different markets have been of great importance in the study of financial markets. The empirical characterization of stochastic processes usually requires the study of temporal correlations and the determination of asymptotic probability density functions *(pdf)*. The first model that describes the evolution of option prices is the Brownian motion. This model assumes that the increment of the logarithm of prices follows a diffusive process with Gaussian distribution [12]. However, the empirical study of temporal series of some of the most important indices shows that in short time intervals the associated *pdf's* have greater kurtosis than a Gaussian distribution [5]. The first step to explain this behavior was done in 1963 by Mandelbrot [13]. He developed a model for the evolution of cotton prices by a stable stochastic non-Gaussian Levy process; these types of non Gaussian processes were first introduced and studied by Levy [14]. However, these distributions are not appropriate for working in long-range correlation scales. These problems can be avoided considering that the temporal evolution of financial markets is described by a Truncated Levy Flight *(TLF)* [15] or by a Normalized Truncated Levy Flight [9].

The other major problem encountered in the analysis of the behavior of different time-series data is the existence of long-term or short-term correlations in the behavior of financial markets (established versus emerging markets [16], developed countries' market indices [1-5], Bombay stock exchange index [17], and Latin American indices [18] and the references therein). Studies that focus on particular country indices [17, 18, 19] generally show that a long term memory effect exists.

The main interest in this work is to compare the international stock market indices and other well-renowned market indices such as the S&P 500. Specifically, this paper seeks to determine whether long memory effects are also present in well diversified international market indices; by testing the financial time-series data of these indices with the Normalized Truncated Levy Flight we wish to check whether the evolution of these indices is explained by the *TLF*.

Previous literature has concluded that the time series of financial indices are explained by the *TLF* model [15, 18, 19]. The Re-Scaled Range Analysis (R/S) and Detrended Fluctuation Analysis (DFA) methods are used to investigate long-range correlations. Previous work has shown that both methods are very powerful for characterizing fractional behavior (see for example [17-21]). As the time-series data for the indices are very small, and the exponents calculated could serve as verification and comparison of the results, both methods are used.

Based on our results we may conclude that using Truncated Levy Flight model is an important and useful tool in the analysis of long memory of time series. In many cases TLF model fits the data very well. However, for a further clarification of the image depicted its analysis should be complemented with the R/S and DFA methods since in many cases these approaches bring new facts into the picture.

2. The Truncated Levy Flight

Levy [22] and Khintchine [23] solved the problem of the determination of the functional form that all the stable distributions must follow. They found that the most general representation is through the characteristic function $\varphi(q)$, by the following equation:

$$\ln(\varphi(q)) = \begin{cases} i\mu q - \gamma|q|^{\alpha}\left[1 - i\beta\dfrac{q}{|q|}\tan(\dfrac{\pi}{2}\alpha)\right] & (\alpha \neq 1) \\ i\mu q - \gamma|q|\left[1 + i\beta\dfrac{q}{|q|}\dfrac{2}{\pi}\ln|q|\right] & (\alpha = 1) \end{cases} \quad (1)$$

where $0 < \alpha \leq 2$, γ is a positive scale factor, μ is a real number and β is an asymmetry parameter that takes values in the interval $[-1,1]$.

The analytic form for a stable Levy distribution is known only in these cases:

$\alpha = 1/2, \beta = 1$, (Levy-Smirnov distribution)

$\alpha = 1, \beta = 0$ α = 1, (Lorentz distribution)

$\alpha = 2$, (Gaussian distribution)

In this work symmetric distributions ($\beta = 0$) with zero mean value ($\mu = 0$) are considered. In this case the characteristic function takes the form:

$$\varphi(q) = e^{-\gamma|q|^{\alpha}} \quad (2)$$

As the characteristic function of a distribution is its Fourier transform, the stable distribution of index α and scale factor γ is

$$P_L(x) \equiv \frac{1}{\pi}\int_0^{\infty} e^{-\gamma|q|^{\alpha}} \cos(qx)dq \quad (3)$$

The asymptotic behavior of the distribution for large values of the absolute value of x is given by:

$$P_L(|x|) \approx \frac{\gamma\Gamma(1+\alpha)sin(\pi\alpha/2)}{\pi|x|^{1+\alpha}} \approx |x|^{-(1+\alpha)} \qquad (4)$$

and the value in zero $P_L(x=0)$ by:

$$P_L(x=0) = \frac{\Gamma(1/\alpha)}{\pi\alpha\gamma^{1/\alpha}} \qquad (5)$$

The fact that the asymptotic behavior for huge values of x is a power law has as a consequence that the stable Levy processes have infinite variance. To avoid the problems arising in the infinite second moment Mantegna and Stanley [15] considered a stochastic process with finite variance that follows scale relations called Truncated Levy Flight (*TLF*). The *TLF* distribution is defined by:

$$\mathrm{P(x)} = \begin{cases} 0 & \mathrm{x} > l \\ c\mathrm{P_L(x)} & -l < \mathrm{x} < l \\ 0 & \mathrm{x} < -l \end{cases} \qquad (6)$$

where $P_L(x)$ is a symmetric Levy distribution and c is a normalization constant. The only stable distributions are the Levy distributions. The *TLF* distribution is not stable, but it has finite variance, thus independent variables from this distribution satisfy a regular Central Limit Theorem. However, depending on the size of the parameter *l* (the cutoff length) the convergence may be very slow [24]. If the parameter *l* is small (so that the convergence is fast) the cut that it presents in its tails is very abrupt. In order to have continuous tails, Koponen [25] considered a *TLF* in which the cut function is a decreasing exponential characterized by a parameter *l*. The characteristic function of this distribution is defined as:

$$\varphi(q) = \exp\left\{c_0 - c_1 \frac{(q^2 + 1/l^2)^{\alpha/2}}{\cos(\pi\alpha/2)} \cos\left[\alpha \arctan(l|q|)\right]\right\} \qquad (7)$$

with c_1 a scale factor and

$$\mathrm{c}_0 \equiv \frac{l^{-\alpha}}{\cos(\pi\alpha/2)} \qquad (8)$$

If one discretizes the time interval with steps Δt, it is found that $T = N\Delta t$. At the end of each interval one must calculate the sum of N stochastic variables that are independent and identically distributed. The new characteristic function will be:

$$\varphi(q,N) = \exp\left\{ c_0 N - c_1 \frac{N(q^2 + 1/l^2)^{\alpha/2}}{\cos(\pi\alpha/2)} \cos\left[\alpha \arctan(l|q|)\right] \right\} \quad (9)$$

For small values of N the return probability will be very similar to the stable Levy distribution:

$$P_L(x=0) = \frac{\Gamma(1/\alpha)}{\pi\alpha(\gamma N)^{1/\alpha}} \quad (10)$$

We note that an alternative way to deal with the convergence of the distribution to a Gaussian in the sense of Central Limit Theorem is to use a Scale-invariant Truncated Lévy process (*STL*) as in [26]. This process uses correlated increments and exhibits Lévy type stability for the increments. We do not use this process in estimation.

Instead, we use in this work a normalized Truncated Lévy Flight model. There are two major advantages to normalize the *TLF* model. First, since it is accepted that the volatility of the return of a financial index is largely proportional to the time scale, normalization over variance allows us to directly compare the statistical properties under different time frames. Second, different markets usually have different risks. This is particularly true when one compares a well-developed market with an emerging market. Normalization essentially implements risk-adjustment, which allows us to compare the behaviors across both developed and emergent markets.

In the Koponen's model the variance can be calculated from the characteristic function:

$$\sigma^2(t) = -\left.\frac{\partial^2 \varphi(q)}{\partial q^2}\right|_{q=0} = t\left[\frac{2A\pi(1-\alpha)}{\Gamma(\alpha)\sin(\pi\alpha)}\right] l^{2-\alpha} \quad (11)$$

In order to normalize Koponen's model to describe the normalized returns of the indices, given that

$$\sigma^2 = -\left.\frac{\partial^2 \varphi(q)}{\partial q^2}\right|_{q=0} \quad (12)$$

It follows that

$$-\left.\frac{\partial^2 \varphi(q/\sigma)}{\partial q^2}\right|_{q=0} = -\frac{1}{\sigma^2}\left.\frac{\partial^2 \varphi(q)}{\partial q^2}\right|_{q=0} = 1 \tag{13}$$

Therefore, by performing a change of variable, a normalized model with characteristic function $\varphi_{NL}(q)$ and volatility 1 can be obtained as:

$$\begin{aligned}
\ln \varphi_{NL}(q) &= \ln \varphi\left(\frac{q}{\sigma}\right) \\
&= c_0 - c_1 \frac{\left((q/\sigma)^2 + 1/l^2\right)^{\alpha/2}}{\cos(\pi\alpha/2)} \cos\left[\alpha \arctan\left(l\frac{|q|}{\sigma}\right)\right] \\
&= \frac{2\pi A l^{-\alpha} t}{\alpha\Gamma(\alpha)\sin(\pi\alpha)}\left(1 - \left(\left(\frac{ql}{\sigma}\right)^2 + 1\right)^{\alpha/2} \cos\left(\alpha \arctan\left(\frac{ql}{\sigma}\right)\right)\right)
\end{aligned} \tag{14}$$

This is the normalized Levy model that will be used for the numerical analysis. This model has been previously used in [9, 19].

To simulate the normalized Truncated Levy model, a Matlab module was developed. The parameter l is fixed at 1 and then parameter A and the characteristic exponent α are adjusted simultaneously in order to fit the cumulative function of the observed returns. On the same grid the cumulative distribution of the observed and simulated returns are plotted for different time lags T in order to visualize how good the fitting is. Time lag T =1 means the returns are calculated by using two consecutive observations; for a general T, the returns are calculated by using: $r_{t=} \log(X_t/X_{t-T})$.

The reader unfamiliar with the α-stable Levy processes (also called Levy flight) may wonder how they can have independent increments and yet be designated as long memory processes. Indeed, this is the case for these processes due to the fact that the increments are heavy tailed. For a detailed discussion please consult Chapter III in [27]. In fact the parameter α of the Levy distribution is inversely proportional to the Hurst parameter. The Hurst parameter is an indicator of the memory effects coming from the fractional Brownian motion which has correlated increments. Furthermore, the Truncated

Levy Flight maintains statistical properties which are indistinguishable from the Levy Flights [24].

3. Rescaled Range Analysis

Hurst [28] initially developed the Rescaled range analysis (R/S analysis). He observed many natural phenomena that followed a biased random walk, i.e., every phenomenon showed a pattern. He measured the trend using an exponent now called the Hurst exponent. Mandelbrot [29, 30] later introduced a generalized form of the Brownian motion model, the fractional Brownian motion to model the Hurst effect.

The complete numerical procedure to calculate the Hurst exponent H by using the R/S analysis is described in detail in [31].
The final relation used to compute the Hurst exponent H is:

$$\log\left(R/S\right) = H\log n + H\log c$$

In this equation *n* is the length of the subseries used, *R/S* is the value of Range/Sample standard deviation statistic within the subseries and *c* is a constant. An ordinary least squares regression is performed using $\log(\frac{R}{S})$ as a dependent variable and $\log n$ as an independent variable. The slope of the equation provides the estimate of the Hurst exponent H. Note that *H log c* is just the intercept in the regression relation.

If the Hurst exponent H for the investigated time series is 0.5, then it implies that the time series follows a random walk which is an independent process. For data series with long memory effects, H would lie between 0.5 and 1. It suggests all the elements of the observation are dependent. This means that what happens now would have an impact on the future. Time series that exhibit this property are called persistent time series and this character enables prediction of any time series as it shows a trend. If H lies between 0 and 0.5, it implies that the time-series possess anti-persistent behavior (negative autocorrelation).

4. Detrended Fluctuation Analysis

The DFA method is an important technique in revealing long range correlations in non-stationary time series. This method was developed by Peng [20, 21], and has been successfully applied to the study of cloud breaking [32], Latin-American market Indices [18], DNA [21, 33, 34], cardiac dynamics [20, 35], climatic studies [36, 37], solid state physics [38, 39], and economic time series [40-42].

The numerical procedure that is used to calculate the DFA exponent α, by using the R/S analysis, is described in detail in [31].

For data series with no correlations or short-range correlation, α is expected to be 0.5. For data series with long-range power law correlations, α would lie between 0.5 and 1 and for power law anti-correlations; α would lie between 0 and 0.5. This method was used to measure correlations in financial series of high frequencies and in the daily evolution of some of the most relevant indices.

4. Data

We studied the behavior of well renowned international market indices: iShares MSCI EAFE Index and the iShares MSCI Emerging Markets Index. We mention a previous study of long memory behavior in some Eastern European economies transitioning to EU [43].

4.1 MSCI EAFE

The MSCI EAFE is a stock index of foreign stocks maintained by Morgan Stanley Capital International. The index includes stocks from 21 developed countries excluding the US and Canada; it is considered as one of the most prominent foreign stock funds benchmark.

4.2 MSCI Emerging Markets Index

The MSCI Emerging Markets Index is a market capitalization index that measures the equity market performance in global emerging markets. It is also maintained by Morgan Stanley Capital International.

Daily closing values of these indices are considered for this study: iShares MSCI EAFE Index (EFA), from August 27, 2001 to May 1, 2009 (1,930 data points) and the iShares MSCI Emerging Markets Index (EEM), from April 15, 2003 to May 1, 2009 (1523 data points). We also used the data of the S&P 500, the New York Stock Exchange index from August 27, 2001 to May 1, 2007 (1,930 data points).

5. Results and Discussions

Hurst as well as DFA analysis is performed to find the persistence of long correlations. Table 1 presents the results of unit root stationarity tests, we refer to [31] for a discussion of this method. Table 2 presents the estimated Hurst and the DFA parameters for the entire respective period. The Hurst exponent and the alpha values obtained are significantly greater than 0.5, thus implying the existence of long term correlations in the financial time-series of all the indices analyzed. The values obtained for the Hurst parameters for the three indices are not significantly different. The values obtained for the DFA alphas are not significantly different between the EEM and S&P500 or between EFA and S&P500. The difference is significant between the EFA and EEM indices.

We note from Table 2 that the value range of the exponents of the EFA and the EEM index are similar with the values obtained for S&P 500. This does not necessarily mean that the extent of the memory effects is the same for all these indices. Indeed, the EFA and EEM indices include stocks from different countries and cannot be expected to move in the same direction as the US Stocks that influence the S&P 500. Furthermore, as we can see in figures 1-3, the pattern of the correlations in EFA and EEM is less stable compared to the pattern characterizing the S&P500. This may be attributed to the fact that the international indices encompass a widely diversified set of stocks from various countries over the world. This raises a question relevant to an investor who looks for the best portfolio profitability: is it better to invest in the US market only, or should an investor look for opportunities in external markets?

When considering long term investments, investors can focus in analyzing the long memory effects to construct a well-diversified, low volatility portfolio. In general it is thought that by diversifying a portfolio in general its variance is reduced. That is only true for investments that behave relatively independently from the current assets already existing in the portfolio. This is why it is crucial to establish the asset's behavior before adding it to a portfolio. From figures 1 through 3, it seems that the memory effects are entirely different between the S&P 500 and the international indices. Specifically, we see that the DFA analysis for the two international indices is much weaker than the analysis for the S&P500. Thus, by diversification into these markets a portfolio manager could potentially lower the variability of the portfolio.

To shed light onto this issue we divide the available data by years and perform the analysis separately within each year. Tables 3 to 5 present the results obtained for each index and for each year available to study. We also present the results of estimating the Levy Flight parameter. We recall that a value close to 2.0 indicates Gaussian behavior of the daily return. We do not present in detail the plots similar to figures 1-3 due to the lack of space in this article, they are available on the article's accompanying webpage:
http://www.math.stevens.edu/~ifloresc/indicesAnalysis.html

There are several things that jump into mind once we study these tables. First and the most important, Hurst parameter analysis gives different results from either DFA analysis or Levy Flight analysis. Specifically, during the years where both DFA and TLF methods detect strong departures from normality (2002 and 2008) *R/S* estimation does not seem to behave any differently than the previous years (Tables 3-5). This may be explained by the fact that the Hurst parameter estimation works best when the data is stationary. While we have tested for unit root non-stationarity in Table 1 and that hypothesis was rejected we cannot guarantee that the data does not possess other types of non-stationarity. There is currently no other type of non-stationarity that may be tested.

Second, looking at several years for which we have an idea of what the results should look like we notice that our hypotheses were confirmed. Specifically, in 2001 and partially in 2002 we expect to see high parameter values due to the end of the dot-com bubble. Again in 2008 we expect to see high values due to the housing crisis and the subsequent market crash in September 2008. Surprisingly, the most reliable confirmation is in the Levy Flight parameter values, they are all much smaller than 2.0 “the normal behavior” parameter value.

Third, recall that one of the purposes of the study was to see if we could indicate to the potential investors the markets which possess the greatest opportunities for diversification. Analyzing the values obtained for the three indices we can see that the behavior of EFA and S&P 500 indices are somewhat similar. We are concentrating the analysis toward the crisis years which are the times when potential investors are faced with the critical investment issue. On the other hand it seems that EEM is representative of markets where investments opportunities are presenting themselves during those times of crisis. We point the reader to the crisis years (2008) and especially to the years after the crisis (2003 and 2009) in table 4. All the indicators show a more pronounced closeness to the normal distribution than the indicators for the other two indices. Table 6 displaying results for normality tests confirm this observation as well. This is surprising since EEM is an emerging market index and those markets are traditionally regarded as hit the most during crisis periods. Additionally, recall that the analysis is done only using only partial data for the year 2009 so even though these conclusions may be evident at the time of publication they were not so when the analysis was performed. In any case the conclusion we draw is that the investment opportunities in these emerging markets are *different,* not necessarily better during crisis periods.

Fourth, the Hurst and DFA estimation for the years 2003 and 2009 and in 2006 for the S&P500 alone give contradictory results. Case in point, the DFA analysis for these years reports anti-persistent behavior while the R/S analysis reports persistent behavior. For further clarification we fit Gaussian distributions for these years to see if the hypothesis of normality can be rejected. The results are presented in Table 6 and the relevant numbers are in bold. The p-values should be read while keeping in mind that they are obtained under the hypothesis that the observations are iid (which is not the case with the rest of the tests) and they are presented only for information purposes. We see that for 2003 the return behavior was that of a Gaussian process. However, for S&P500 in 2006 the normality hypothesis is rejected. In fact it seems that the results parallel the conclusions drawn following the Levy Flight parameter estimation. For a wealth of analysis and complete results we again direct the interested reader to the above mentioned webpage.

Finally, we want to point out the analysis for the year 2009, the current year of this article, and the year after the great crisis of 2008. The analysis only contains the first 7 months of this year (we are in August at the date of writing this article). Normality hypothesis is rejected for S&P500 and the EFA index but cannot be rejected for the EEM index. We see the same dichotomy pointed out earlier in the behavior of the three indices. The analysis seems to indicate that while the S&P500 and the EFA indices may still possess long memory behavior, the EEM index already returned to the normal pre-crisis behavior.

In conclusion by fitting the three different methods to the data under study we found the behavior we expected to see reflected best in the Truncated Levy Flight analysis. However, we also discovered that DFA and R/S methods provided complementary answers and ideas to the ones already provided by the TLF analysis.

Additionally, we find that the behavior of the three indices under study is different during the crisis period, but mostly after a crisis period. An investor would be served well by searching for investment opportunities and switching his/her investments when the crisis is coming. Diversifying into emerging markets during and especially after a crisis periods seems to be the right way to proceed.

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Table 1: P-values for the unit root stationarity test.

Index	PP	ADF	KPSS
EAFE Index (EFA)	<0.01	<0.01	>0.1
Emerging Markets Index (EEM)	<0.01	<0.01	>0.1
S&P 500 (SP500)	<0.01	<0.01	>0.1

DFA= Detrended Fluctuation Analysis Hurst= Rescale Range Analysis,
ADF= Augmented Dickey-Fuller Test for unit root stationarity, PP= Phillips-Perron Unit Root Test,
KPSS= Kwiatkowski-Phillips-Schmidt-Shin Test for unit root Stationarity.

For the ADF and the PP the null hypothesis is the unit root non-stationarity, and for the KPSS the null hypothesis is stationarity. The tests reject the unit-root type nonstationarity. So it is reasonable to assume that the data does not possess evidence of unit root nonstationarity.

Table 2: Values of exponent α and H for all indices calculated using DFA Method and R/S analysis respectively. The entire period available was used for each index

Index	DFA Exponent		Hurst Exponent	
	A	error	H	error
EAFE Index (EFA)	0.6067	0.0220	0.5761	0.0053
Emerging Markets Index (EEM)	0.743	0.0390	0.5779	0.0047
S&P 500	0.6707	0.0202	0.5665	0.0041

Table 3: Values of DFA exponent α, Hurst exponent H, and Levy Flight parameter α for the EAFE Index (EFA) year by year.

EAFE Index (EFA)	DFA		Hurst		Levy Flight parameter (α)			
	A	error	H	error	α@T=1	α@T=4	α@T=8	α@T=16
2001	0.8173	0.0282	0.8098	0.0218	1.70	1.40	1.40	1.60
2002	0.6844	0.0156	0.5929	0.0088	1.80	1.70	1.60	2.00
2003	0.5066	0.0111	0.5526	0.0134	1.80	1.90	1.70	1.60
2004	0.6186	0.0130	0.5534	0.0095	1.85	1.70	1.70	2.00
2005	0.6098	0.0204	0.5593	0.0114	2.00	1.80	1.99	2.00
2006	0.6016	0.0111	0.5750	0.0128	1.60	1.40	1.40	1.40
2007	0.6300	0.0119	0.5504	0.0092	1.80	1.80	1.90	1.90
2008	0.7870	0.0290	0.5484	0.0084	1.30	1.30	130	1.30
2009	0.4877	0.0157	0.5742	0.0134	1.70	1.90	1.90	1.90

Table 4: Values of DFA exponent α, Hurst exponent H, and Levy Flight parameter α for the Emerging Markets Index (EEM) year by year.

Emerging Markets Index (EEM)	DFA		Hurst		Levy Flight parameter (α)			
	α	Error	H	Error	α@T=1	α@T=4	α@T=8	α@T=16
2003	0.4474	0.0123	0.5835	0.0113	2.00	2.00	2.00	2.00
2004	0.6970	0.0229	0.6264	0.0074	1.70	1.60	1.60	1.60
2005	0.6064	0.0202	0.6203	0.0079	1.80	1.75	1.70	1.70
2006	0.6373	0.0196	0.5300	0.0126	1.50	1.60	1.50	1.40
2007	0.6657	0.0138	0.5318	0.0109	1.70	1.80	1.70	1.80
2008	0.5546	0.0448	0.5236	0.0102	1.40	1.40	1.40	1.40
2009	0.3866	0.0120	0.5605	0.0190	1.85	1.99	2.00	2.00

Table 5: Values of the DFA exponent α, Hurst exponent H, and Levy Flight parameter α for the S&P500 Index year by year.

S&P 500 Index	DFA		Hurst		Levy Flight parameter (α)			
	α	Error	H	Error	α @T=1	α @T=4	α @T=8	α @T=16
2001	0.6888	0.0188	0.7836	0.0152	1.40	1.30	1.30	1.30
2002	0.7434	0.0189	0.5865	0.0098	1.50	1.40	1.50	1.60
2003	0.4979	0.0114	0.5702	0.0082	1.50	1.60	1.50	1.50
2004	0.5199	0.0124	0.6080	0.0070	1.90	1.80	1.90	2.00
2005	0.6674	0.0132	0.5589	0.0086	1.90	1.80	1.90	2.00
2006	0.4745	0.0131	0.5170	0.0135	1.60	1.99	1.60	1.70
2007	0.6574	0.0185	0.5220	0.0104	1.60	1.80	1.70	2.00
2008	0.7677	0.0287	0.5168	0.0097	1.40	1.40	1.40	1.40
2009	0.4449	0.0159	0.5639	0.0150	1.50	1.80	1.90	1.50

Table 6: Goodness of fit p-values for fitting the Gaussian distribution to the return data of the three indices

	EFA Index	EEM Index	S&P 500 Index
2001	0.023		0.303
2002	0.074		0.086
2003	**0.283**	**0.668**	**0.327**
2004	0.138	0.060	0.026
2005	<0.005	0.031	0.133
2006	<0.005	0.013	**<0.005**
2007	<0.005	<0.005	<0.005
2008	<0.005	<0.005	<0.005
2009	**0.035**	**0.100**	**0.033**

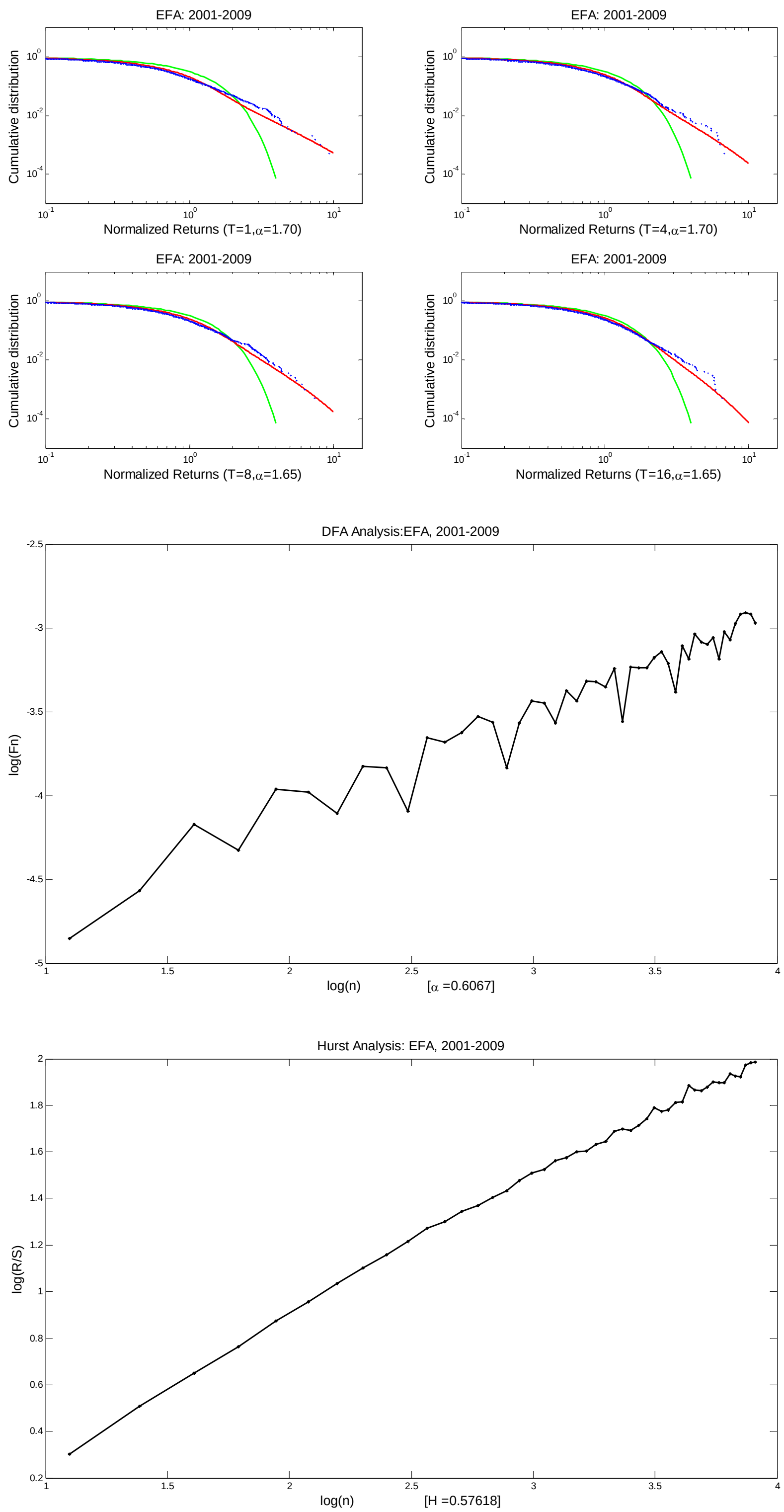

Figure 1: Analysis results for EFA index using the entire period available. Top 4 images: Fitting using TLF. Blue points represent the empirical distribution; red curve the best fitted TLF distribution; green curve best Gaussian curve fit. There are four fitted distribution one for each return lag *T* considered. Middle image: results for the DFA analysis. Points should be close to a line, the slope of the line is the DFA parameter (α). Bottom image: results for the *R/S* analysis. Points should be close to a line, the slope is the Hurst parameter (*H*).

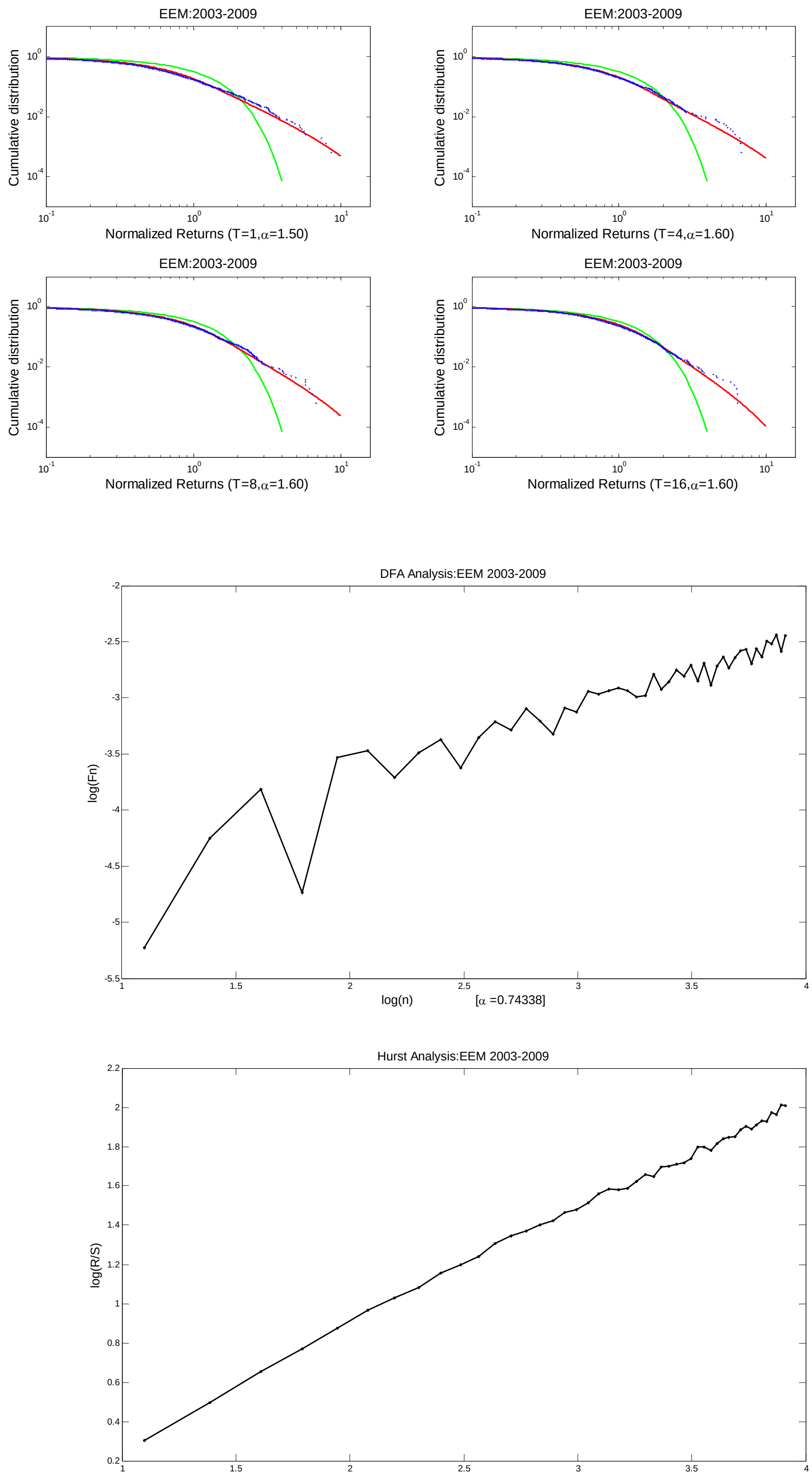

Figure 2: Analysis results for EEM index using the entire period available.

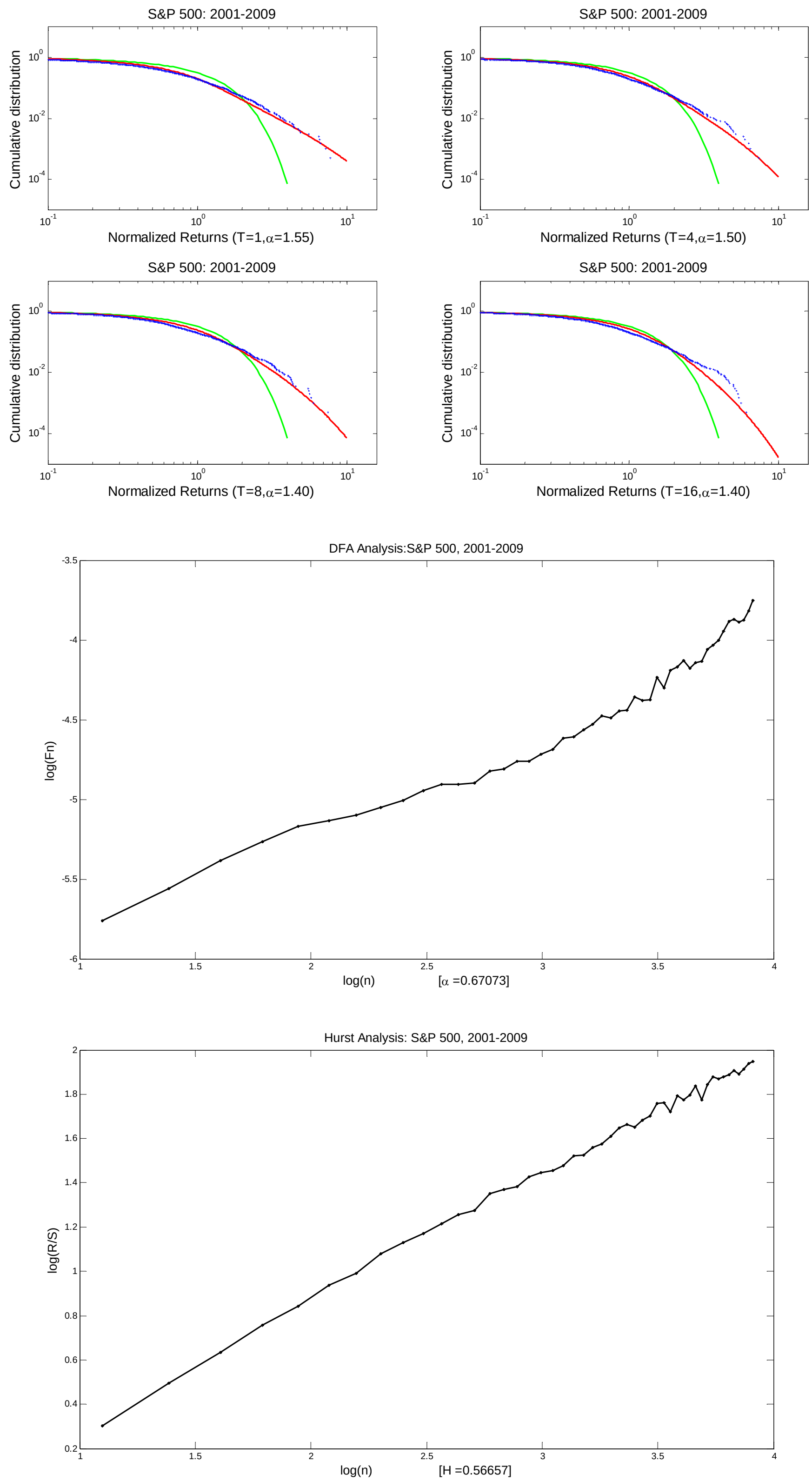

Figure 3: Analysis results for S&P 500 index using the entire period available.

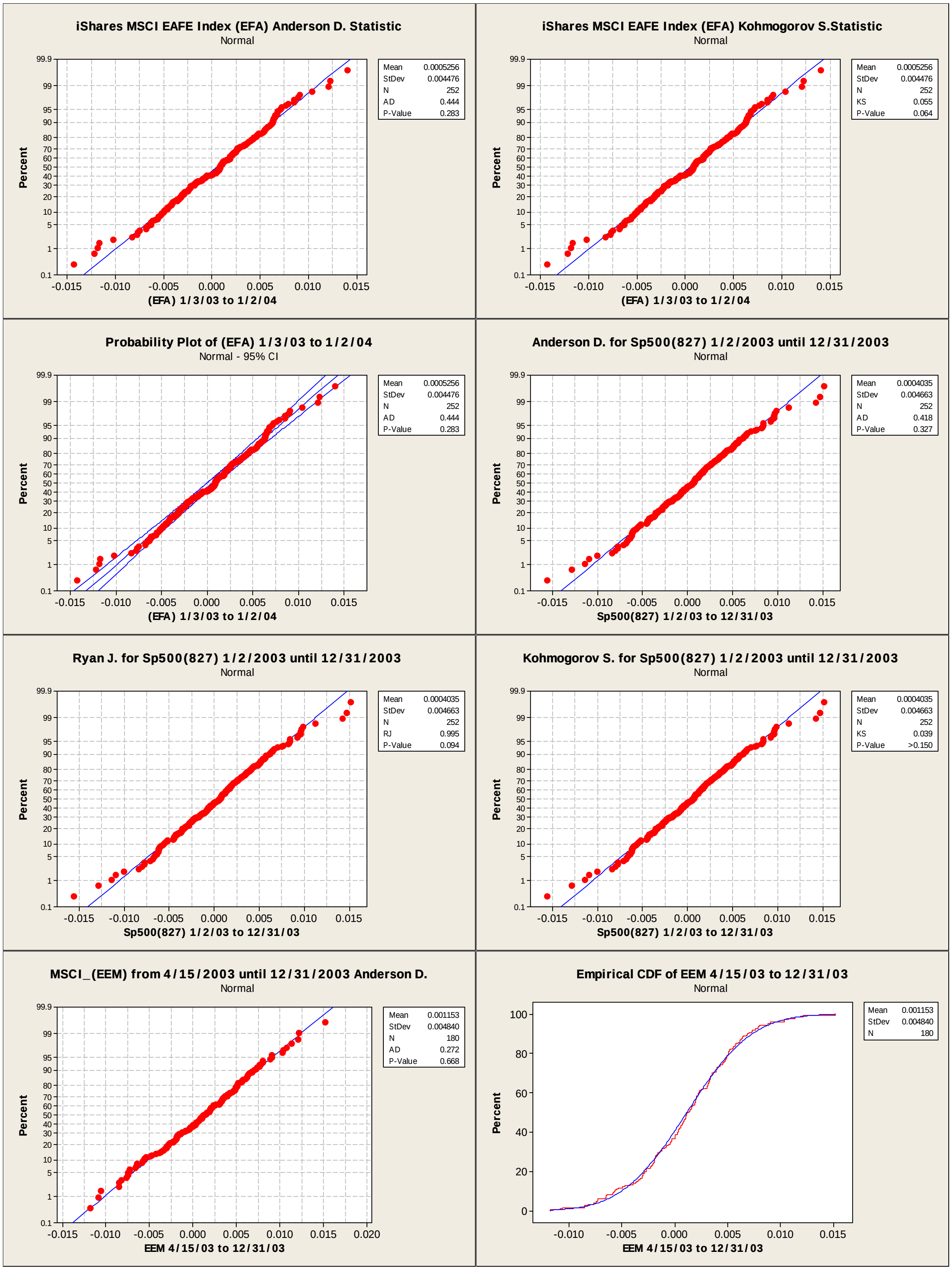

Figure 4: Several normality tests for 2003 using the three indices