

Midterm 1

Math 611 Probability

November 14, 2005
to be handed in on November 21, 2005

- (1) Ali Baba is caught by the sultan while stealing his daughter. The sultan is being gentle with him and he offers Ali Baba a chance to regain his liberty.

There are 2 urns and m white balls and n black balls. Ali Baba has to put the balls in the 2 urns however he likes such that no urn is empty. After that the sultan will chose an urn at random then pick a ball from that urn. If the chosen ball is white Ali Baba is free to go, otherwise Ali Baba's head will be at the same level as his legs.

How should Ali Baba divide the balls to maximize his chance of survival?

- (2) Choose a point A at random in the interval $[0, 1]$. Let L_1 (respectively L_2) be the length of the bigger (respectively smaller) segment determined by A on $[0, 1]$. Calculate:

(a) $\mathbf{P}(L_1 \leq x)$ for $x \in \mathbb{R}$.

(b) $\mathbf{P}(L_2 \leq x)$ for $x \in \mathbb{R}$.

- (3) Two friends decide to meet at the Castle gate of Stevens Institute. They each arrive at that spot at some random time between a and $a + T$. They each wait for 15 minutes then leave if the other did not appear. What is the probability that they meet?

- (4) We know that the random variables X and Y have joint density $f(x, y)$. Assume that $\mathbf{P}(Y = 0) = 0$. Find the densities of the following variables:
- (a) $X + Y$
 - (b) $X - Y$
 - (c) XY
 - (d) $\frac{X}{Y}$
- (5) Give a counterexample to the statement $\mathbf{E}(XY) = \mathbf{E}(X)\mathbf{E}(Y)$ implies that X and Y are independent.