

# List of Subjects for Midterm 1

## *Math 611 Probability*

October 29, 2005

- (1) Measure (Probability) Spaces.
  - (a)  $\sigma$ -Algebras
  - (b) Measurable spaces, Probability spaces
  - (c) Lebesgue Measure, Properties
- (2) Probability Measure.
  - (a) Conditional Probability, Bayes Rule. Independence.
  - (b) Borel Cantelli lemmas. Kolmogorov's zero-one law
- (3) Random Variables (Measurable functions).
  - (a) Distribution Function. Indicators.
  - (b) Discrete and continuous random variables. Distribution classes (e.g., Dirac  $\delta$ , Uniform, Binomial etc.)
- (4) Integration Theory.
  - (a) Construction of the integral. Monotone convergence theorem. The standard argument.
  - (b) Fatou lemma, Dominated convergence, Jensen's inequality, Markov inequality, Cauchy-Schwartz inequality.
  - (c)  $L^p$ -spaces. Definition.
  - (d) Expectation. Variance and correlation coefficient.  $p$ -th moment and central  $p$ -th moment.

- (5) Functions of random variables.
  - (a) Distribution of functions of random variables.
  - (b) The Transport Formula, the Jacobian-transport formula.
  - (c) The Law of Large numbers
- (6) Product Spaces. Joint distribution. Sums of random variables.
  - (a) Product Spaces. Fubini Theorem.
  - (b) Sums of random variables. Convolutions.
  - (c) The de Moivre-Laplace Theorem. Stirling's formula.
  - (d) Conditional Distribution. Conditional Expectation.
- (7) Generating Functions.
  - (a) Regular and Probability Generating functions.
  - (b) Characteristic function. (Moment generating function).

For problems look at the exercises in the lecture notes, the first assignment, and the problems in the textbook (both solved and proposed). Also, if you wish, you can try solving problems from any probability book. Remember, you can always come and see me to talk about such problems.