

EXAM 1 (SAMPLE)  
*Ma 623 Stochastic Processes*

April 3, 2006

- (1) Mary has \$20 when she walks into a casino in Las Vegas. She will make \$1 bets at the Blackjack table until she goes broke. Bets are independent, and on each bet the probability is 0.49 that she wins \$1 and 0.51 that she loses \$1.

What is the expected number of bets she will make before going broke.  
(You can assume this expected number is finite)

- (2) Trucks arrive at a UPS station according to a renewal process with  $U(0, 1)$  (in hours) interarrival times. All packages waiting at that station are instantly loaded as soon as a truck arrives. Packages arrive at the UPS station according to a Poisson process with rate 4/hour. Calculate:

$$\lim_{t \rightarrow \infty} \mathbf{P}\{\text{NO packages at the station at time } t\}$$

- (3) Consider the Markov Chain on  $\mathcal{S} = \{1, 2, 3\}$  described by:

$$\mathbf{P} = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \\ 1/4 & 1/4 & 1/2 \end{pmatrix}$$

- a) Is this Markov Chain recurrent? Prove your answer.  
b) Find the stationary distribution  
c) Find  $\lim_{n \rightarrow \infty} \mathbf{P}\{X_n = X_{n+1} | X_0 = 1\}$
- (4) For the Markov Chain on state space  $\mathcal{S} = \{0, 1, 2, 3, 4, 5\}$  and transition matrix:

$$\mathbf{P} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7/8 & 0 & 1/8 \\ 0 & 0 & 3/4 & 0 & 1/4 & 0 \\ 0 & 1/10 & 0 & 3/10 & 0 & 6/10 \\ 0 & 0 & 1/5 & 0 & 4/5 & 0 \end{pmatrix}$$

- a) Determine which states are positive recurrent, null recurrent, transient

b) The period of all states

c)

$$\lim_{n \rightarrow \infty} \mathbf{P}\{X_n = 0 | X_0 = i\}, \quad i = 0, 1, 2, 3, 4, 5.$$