# Homework 6

*Ma 623 Stochastic Processes*

due Tuesday April 11 2006

From Ross "Stochastic Processes" 2nd ed. do the following:
page *219* exercises *4.4, 4.5, 4.8, 4.12, 4.18*

In addition do the following problems:

(I) Give an example of a Markov Chain with more than one stationary distribution.

(II) Let $X_k$ be an irreducible Markov chain on $\mathbb{Z}$ with invariant initial distribution. Let $p_k(n) = Pr\{X_n = k\}$, and suppose that the limit $p_k(\infty) = \lim_{n\to\infty} p_k(n)$ exists.

a) Check that the limiting probabilities satisfy the stationarity equations.

b) Let $P = \begin{pmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix}$. Find the initial distribution of $X_0$ that results in a stationary process. Then, find the limiting distribution $\lim_{n\to\infty} p_k(n)$, say $k \in \{0, 1\}$.

c) Let $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the initial distribution of $X_0$ that results in a stationary process. Explain why $\lim_{n\to\infty} p_k(n)$ does not exist.

d) Let $P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Find the initial distribution of $X_0$ that results in a stationary process. Then, find the limiting distribution $\lim_{n\to\infty} p_k(n)$.

(III) Suppose $X$ is a MC with transition matrix: $P = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$. Show that $Y_n = (X_n, X_{n+1})$ is also a Markov chain. Find its transition probability and the stationary distribution (if it exists).

(IV) *(SIMULATION PROBLEM). There are many theoretical results for Markov Chains, however in many cases simulation is the most expedient way to study them.*

Suppose there are $n$ people on a Stevens committee discussing the heating issue in the Kidde building. Assume that every time one speaker finishes, one of the other $n-1$ speakers are equally likely to continue the debate. Further assume that each person speaks an exponential amount of time with parameter $\lambda$. How long does it take on average for all the members of the committee to take part in the discussion.