Homework 5
Math 611 Probability

due Monday Nov. 27 2006 in class

From your textbook do the following exercises:

Chapter 3 page 161 Problems 15, 16, 18, 37.

Do example 3.12 on page 109 then do exercise 21 on page 164. After that do exercise 40 on page 168.

Chapter 5 page 330 do problems 12, 21, 22.

In addition do the following problems (featured in last year's take-home Final):

(1) Ann and Bob each attempt 100 basketball free throws. Ann has probability 0.60 of success on each attempt. Bob has probability 0.50 of success on each attempt. The 200 attempts are independent.

What is the approximate numerical probability that Ann and Bob make exactly the same number of free throws?

(2) Let $X_1, X_2, \ldots$ be independent random variables each of which is uniformly distributed on $(0, 1)$. Let N_n be the number of $X_1, X_2, \ldots, X_n$ which are less than or equal $1/2$. Show that:

$$\sqrt{n}\left(2\frac{N_n}{n} - 1\right) \xrightarrow{d} N(0, 1),$$

where $\xrightarrow{d}$ denotes convergence in distribution.

(3) A model for the count of tumors detected in rats exposed to a carcinogen in an experiment assumes:

i. The number M of tumors initiated is a random variable with mean μ and standard deviation σ.

ii. All tumors start at time $t = 0$. The times $T_1, T_2, \ldots, T_M$ at which the tumors are detected are iid random variables with cdf $F(t)$.

Please solve the following problems:

(a) Let J_t be the number of tumors detected by time t. Give the mean and variance of J_t in terms of μ, σ, and F.

(b) Show that if M has the Poisson distribution $\mathcal{P}(\lambda)$ with mean λ, then J_t has the $\mathcal{P}(\lambda F(t))$ distribution.