# Homework 6
## *Math 611 Probability*

due Friday Dec. 8 2006 at 3:00pm

From your textbook do the following exercises:

Chapter 5 page 330 Problems 42, 52.
Chapter 9 page 633 Problems 2, 11.

SRW problems:

(1) Consider a symmetric random walk $S$ with $S_0 = 0$. Let $T = \min\{n \geq 1 : S_n = 0\}$ be the time of the first return of the walk to its starting point. Show that:

$$\mathbf{P}(T = 2n) = \frac{1}{2n-1}\binom{2n}{n}2^{-2n},$$

and deduce that $\mathbf{E}T^{\alpha} < \infty$ if and only if $\alpha < \frac{1}{2}$.
(Stirling's formula: $n! \simeq \sqrt{2\pi n}\, n^n e^{-n}$).

(2) Consider a symmetric random walk with an absorbing barrier at $N$ and a reflecting barrier at 0 (when the random walk reaches 0 it moves to 1 at the next step with probability one). Let $\alpha_k(j)$ be the probability that the particle having start at $k$, visits 0 exactly $j$ times before being absorbed at $N$. By convention, if $k = 0$ then the starting point counts as one visit already. Show that:

$$\alpha_k(j) = \frac{N-k}{N^2}\left(1 - \frac{1}{N}\right)^{j-1} \quad , \quad \forall j \geq 1,\ 0 \leq k \leq N$$

In addition do the following problems (featured in last year's take-home Final):

(3) Let $S_n$ be the number of successes in a series of independent trials whose probability of success at the $k^{th}$ trial is $p_k$. Suppose $p_1, p_2, \ldots, p_n$ depend on $n$ in such a way that:

$$p_1 + p_2 + \cdots + p_n = \lambda, \quad \text{for all } n,$$

while $\max\{p_1, p_2, \ldots, p_n\} \to 0$ when $n \to \infty$. Prove that $S_n$ has a Poisson distribution with parameter $\lambda$ in the limit as $n \to \infty$.

(4) Let $\nu$ be the total number of spots which are obtained in 1000 independent rolls of an unbiased die.

(a) Find $\mathbf{E}[\nu]$

(b) Estimate the probability $\mathbf{P}\,(3450 < \nu < 3550)$