

Ma 612 Mathematical Statistics
Final Examination

Wednesday May 9, 2007

- (1) Let X_1, X_2, \dots, X_n be *dependent* Bernoulli random variables such that $P\{X_i = 1\} = \frac{1}{2}$ and

$$P(X_{k+1} = x_k \mid X_1 = x_1, \dots, X_k = x_k) = \theta$$

for all $1 \leq k < n$. Here θ is an unknown parameter.

- (a) Find a minimal sufficient statistic.
 - (b) What is the UMP test of $\theta \leq \frac{1}{2}$ vs. $\theta > \frac{1}{2}$?
 - (c) What is the distribution of the test statistic?
- (2) Let X_1, \dots, X_n be iid $N(\theta, 1)$. Show that the best unbiased estimator of θ^2 is $\bar{X}^2 - 1/n$. Calculate its variance, and show that it is greater than the Rao-Cramér lower bound.
- (3) The random variable X has pdf $f(x) = e^{-x}$, $x > 0$. One observation is obtained on the random variable X^θ , and a test of $H_0 : \theta = 1$ vs. $H_1 : \theta = 2$ needs to be constructed. Find the UMP level $\alpha = 0.05$ test and compute the Type II error probability.
- (4) Do 8.55 on page 413 from your textbook.
- (5) Suppose that X is a random variable from a beta distribution with parameters θ and 1, $\beta(\theta, 1)$. Let x be its observed value.
- (a) Let $Y = -1/\log(X)$. Let y be the corresponding outcome for Y . Evaluate the confidence coefficient of the interval $[y/2, y]$.
 - (b) Find a pivotal quantity and use it to set up a confidence interval with the same confidence coefficient as the interval in part (a).
 - (c) Compare the 2 confidence intervals.