Homework 3

Ma623 Stochastic Processes

due Tuesday March 20 2006

For this assignment please do the following problems from the pages 229-237 (ch 5) of your textbook: page 229 ex. 6, page 230 ex. 8 (alternating renewal process), page 230 ex. 2, page 232 ex. 12.

In addition do the following exercises:

(1) Consider a single-server bank in which potential customers arrive at a Poisson rate λ. However, an arrival only enters the bank if the server is free when he or she arrives. Let G denote the service distribution.

   (a) At what rate do customers enter the bank?

   (b) What fraction of potential customers enter the bank?

   (c) What fraction of the time is the server busy?

   Simulation part Now let us try and use simulation to solve this problem. Assume that $\lambda = 2$ customers per minute, and that $G = Uniform[0, 1]$. Use software to generate the Poisson process of the arrivals and the times of the service (the blackout periods). Now calculate the new arrival process.

   (d) Using the elementary renewal theorem you were able to calculate in part (a) the average rate of the new process when t is large. Now use simulation to do the same thing. Use $t = 10,000$ minutes and as many repetitions as you think necessary.

   (e) Again using the simulation answer parts (b) and (c). Use the same value for t as above.

   (f) Calculate using the theory the answers for the particular case considered in the simulation for your specific values of λ and G. Then record and give the order of difference between the theoretical values and the simulation.

(2) A fair six sided die has sides: $10, 15, 25, 40, 45, 75$. Let S_n be the sum of the first n rolls and $N(t)$ the number of times the die was rolled before reaching the total t.

Calculate:

(a) $\mathbf{P}(S_n = 2,678,495 \text{ for some } n)$

(b) The 95^{th} percentile of $N(2,678,495)$