

Homework 4
Ma623 Stochastic Processes
due Tuesday April 3, 2006

For this assignment please do the following problems:
page 73 (ch 2) problem 1, page 78 problems 9 and 11 (note that the problem 10 has been done in class)
page 108 (ch 3) problem 2.
In addition do the following exercises:

- (I) A fair six sided die has sides: 10, 15, 25, 40, 45, 75. Let S_n be the sum of the first n rolls and $N(t)$ the number of times the die was rolled before reaching the total t .

Calculate:

- (a) $\mathbf{P}(S_n = 2,678,495 \text{ for some } n)$
(b) The 95th percentile of $N(2,678,495)$

- (II) Suppose X is a MC with transition matrix: $P = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$.
Show that $Y_n = (X_n, X_{n+1})$ is also a Markov chain. Find its transition probability and the stationary distribution (if it exists).

- (III) (*SIMULATION PROBLEM*). *There are many theoretical results for Markov Chains, however in many cases simulation is the most expedient way to study them.*

Suppose there are n people on a Stevens committee discussing the heating issue in the Kidde building. Assume that every time one speaker finishes, one of the other $n - 1$ speakers are equally likely to continue the debate. Further, assume that each person speaks an exponential amount of time with parameter λ . How long does it take on average for all the members of the committee to take part in the discussion.