

Wednesday, April 12<sup>th</sup>, 2000

**Practice exam**  
From Spring 2000  
**Midterm 2**

NAME:

**Note:** Questions marked “optional” are worth extra credit.

(6 pts) **1.** If the function  $f(x)$  is given by  $f(x) = \begin{cases} c(x^2 + 1), & \text{for } 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$ , where  $c$  is a constant, the value of  $c$  which makes the above function a PDF is:

A. 12

B.  $\frac{4}{15}$

C.  $\frac{3}{12}$

D.  $\frac{1}{12}$

(4 pts) **2.** If the random variable  $X$  has the CDF  $F(x) = 1 - e^{-2x}$ , for  $0 \leq x \leq \infty$ , the value of  $f(2)$  is:

A. 0.03663

B.  $1 - e^{-4}$

C. 0.2405

D. 0.1901

(6 Pts) **3.** The probability that any one vehicle will turn left at State and Airport Road intersection is 0.2. The left turn lane at this intersection has room for three vehicles. What is the probability that six cars must arrive at the intersection to fill up the left-turn lane?

A.  $\binom{6}{3}$

B. 0.04096

C. 0.01246

D. 0.01178

(4 Pts) **4.** If  $\Gamma(7/3)=1.19$  what is  $\Gamma(1/3)$ ?

A. 1.1906

B. 2.6775

C. 0.8925

D. 1.7724

5. The mean number of telephone calls at a customer service line, arriving in a one-minute interval equals 3. If  $X$  counts how many calls are made in a 3 minute interval find:

(4 Pts) (a) The mean and variance for  $X$ .

(5 Pts) (b) Probability that there are no more than 5 calls in the 3-minute interval, knowing that there are at least 2?

If  $T$  is the time between successive calls find:

(4 Pts) (c) Expected time between successive calls and variance of this time

(5 Pts) (d) If a customer is already on the line and the call takes 4 minutes to complete, what is the probability that exactly 4 other callers are waiting on the line?

(2 Pts) (e) What is the probability that a call takes **exactly** 3 minutes to complete?

6. Daily CPU time used by the Statistics Department at Purdue University is a random variable (measured in hours) with the probability density function:

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x), & \text{for } 0 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

- (7 Pts) (a) Find the expected value and the variance of daily CPU time
- (7 Pts) (b) Find the Cumulative Distribution Function (CDF) of X
- (6 Pts) (c) The CPU time costs the Department \$10 per hour. Find the expected value and the variance of daily cost for CPU time.

7. We know that the yearly profit of a very small brokerage firm is a random variable with the following distribution (in millions of dollars):

$$f(x) = \begin{cases} \frac{4}{27}(3-x)x^2, & \text{for } 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

- (5 Pts) (a) Find the probability that the next year's profit is over \$1,000,000.  
(5 Pts) (b) Knowing that in June they already have \$500,000 profits what is the probability that by the end of the year the total profit will be more than \$2.5 millions?  
(5 Pts) (c) Find the mean and variance of yearly profit.  
(5 Pts) (d) If each year IRS takes tax from profit an amount equal to  $0.1 * X^2 + 0.1$ , how much should they expect to pay next year?

**8.** The scores on an IQ test given to the Stat225 TA's is a random variable normally distributed with mean 115 and standard deviation 10.

(6 Pts) (a) What is the probability that my Stat225 TA is an imbecile (has IQ score less than 100)?

(6 Pts) (b) What is the probability that my Stat225 TA is a genius (has IQ score over 135)?

(8 Pts) (c) I heard that the TA of the other section scored really high; his score puts him at the 93<sup>rd</sup> percentile, that is 93% of the people taking the test have scores smaller than his. What is his score (rounded to the nearest integer)?

**9. (Optional)** Children's heights in Lafayette are approximately normally distributed, with mean 64 inches and standard deviation 12 inches. We randomly pick four children out of the population.

(5 Pts) (a) What is the probability that the first 2 will be taller than average and the last 2 will be shorter than average?

(5 Pts) (b) What is the probability that exactly 2 out of the 4 children will be taller than average?

(5 Pts) (c) What is the probability that the first child chosen will be taller than 6 feet?

(5 Pts) (d) What is the probability that exactly 2 out of the 4 children chosen will be taller than 6 feet?

**NOTE:** Remember to make distinction between feet and inches

*(20 bonus points)*