Practice Exam 2

Name:

- There are 3 problems, each worth between 30 and 40 points.
- Before you start, make sure your exam is not missing any page.
- You may do the problems in any order you like.
- You can earn lots of partial credits if you show your work.
- You are allowed two pages of notes (both sides) and a calculator.

For instructor's use only

Problem	Points	Score
1	30	
2	30	
3	40	
Total	100	

1. A printing money machine from the Romanian National Bank has an electronic device that will ring and stop printing when the first bad bancknote bill is encountered. It is known that on the average it takes about 20 bills to hear the device ringing from the start of the machine (this is a really reliable romanian machine).

   (a) Find the probability that one banknote is printed fine.

   (b) Find the probability that after exactly 10 bills the device will ring

   (c) Now assume that the machine will continue running even after a bad bill. A clerk is supposed to stop the machine when he hears the device ring 3 times. Find the probability that when he will stop the machine there will only be 20 bills produced.

2. The percentage of impurities per batch in a certain type of industrial chemical can be modeled by a random variable X with the following probability density function:

$$f(x) = \begin{cases} 1 - 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

(a) Suppose that a batch with more than 30% impurities cannot be sold. What is the probability that a randomly selected batch cannot be sold because of this reason?

(b) Calculate expected percentage of impurities and the standard deviation of this percentage

(c) Suppose that the value of each batch is give by $V = 500(1 - X)$. Find the expected value and the variance of V.

(d) What percentage of impurities is exceeded in exactly 50% of the batches.

3. We conduct the following experiment: we take a light bulb and keep it lit for all its lifetime. Then as soon as it's done we replace it immediately with another one and so on. It is assumed that the life of a such light bulb is exponentially distributed with variance equal to 10000 (hours squared)

   (a) Find the probability that the next light bulb will last more than 200 hours

   (b) If we look to the number of bulbs replaced over time we have a process called The Renewal process. Let X be the number of light bulbs replaced in a 1000 hours interval. What is the distribution of X (name it and give its parameter(s))

   (c) Referring to the above if we know that in an 100 hour interval exactly one light bulb failed, what is the probability that it lasted for 75 hours before it failed