# Review Problems for Final Exam

1) A continuous random variable X has pdf $f(x)$ and cdf $F(x)$. Identify the statements in the list below that are TRUE. (More than one may be true, identify all of them!)
   a. $F(x)$ cannot have the value 2.5.
   b. $f(x)$ cannot have the value 2.5.
   c. The graph of $f(x)$ cannot have ups and downs.
   d. The graph of $F(x)$ cannot have ups and downs.
   e. The indefinite integral of $F(x)$ is $f(x)$.
   f. The derivative of $f(x)$ is $F(x)$.
   g. $F(x)$ cannot be negative.
   h. $f(x)$ cannot be negative.

2) If X is discrete in the above statement what remains true and what is changed

3) The total score for each student enrolled in a Statistics class is normally distributed. The students are divided into two sections and the instructor in each section may decide on the assignment of grades. In the first section the instructor will follow the following grading scale:

$$\text{Total } > \mu + 2\sigma : \text{A}$$
$$\mu < \text{Total} < \mu + 2\sigma : \text{B}$$
$$\mu - \sigma < \text{Total} < \mu : \text{C}$$
$$\mu - 2\sigma < \text{Total} < \mu - \sigma : \text{D}$$
$$\text{Total } > \mu - 2\sigma : \text{F}$$

The second section instructor will use the following grading scale:

$$\text{Total } > \mu + \sigma : \text{A}$$
$$\mu - \sigma < \text{Total} < \mu + \sigma : \text{B}$$
$$\mu - 2\sigma < \text{Total} < \mu - \sigma : \text{C}$$
$$\text{Total } > \mu - 2\sigma : \text{D}$$

   a. Which section is giving a greater percentage of A's? Explain.
   b. Which section is giving a greater percentage of B's? Explain.

3. The daily rainfall in Hoboken in the month of June averages 5 mm, with a standard deviation of 1 mm. What is the APPROXIMATE probability that the total rainfall in the month of June exceeds 160 mm?

4) We assume that the stock market goes up or down, independently from day to day, with a 75% chance of going up on any particular day. If we observe the stock market on 20 consecutive days, what is the probability that it goes up on 18 or more days?

5) A fair die is rolled twice. Suppose X denotes the sum, and Y denotes the number of distinct faces obtained.

a. Find the joint distribution of X and Y.

b. Find the expected value of Y/X.

c. Find the expected value of X/Y.

d. Is the answer in (c) the reciprocal of the answer in (b)? Should it be?

6) Cars pass a certain point on the I1/I9 highway according to a Poisson distribution with mean 10 per minute.

a. Find the average number of cars that pass by during an hour.

It is known that the time interval between two cars is a random variable having an exponential distribution with mean 1/10 minute (6 seconds)

b. I see a car passing by at exactly 9:00AM. Find the probability that I must wait at least 40 seconds to see the first car.

c. How much do you expect to wait until you see the $6^{th}$ car?

d. If I observe the highway from 9:00 to 9:02 AM in any particular day what is the probability that I see at least 6 cars in this interval? (you may want to use a table or R for this)

e. I observe each day from 9:00 to 9:02 AM for a full week. Find the probability that I will see at least 6 cars at least twice next week.

f. I observe each day from 9:00 to 9:02 AM for a full year. Approximate the probability that I will see at least 6 cars at least 270 times next year.

7) Twenty random numbers from the interval [0,1] are added together. Find an approximation to the probability that the sum is between 5 and 15.

8) If I set the thermostat in my office at temperature T, then the actual temperature is normally distributed with mean T and variance 1. At what temperature should I set the thermostat if I want to keep the temperature above 69 degrees at least 99% of the time?

9) The 10th percentile of a normally distributed random variable is 2 and the 90th percentile is 8. What are the mean and variance of this random variable?

10) Next week Joe will play many tennis matches with Eddie from Ohio (more than 30). However, Joe is not a very good player and we know in advance that the mean and the variance of the number of matches in which Joe will beat Eddie are 2 and 1.2, respectively. What is the probability that Joe will beat Eddie in four or more matches?

11) Find the probability that X is strictly between 0.5 and 5.0 if

a. X has a Binomial(10,0.4) distribution.

b. X has a Poisson(6) distribution.

c. X has a Geometric(0.1) distribution.

d. X has a Negative Binomial(2,0.3) distribution.
e. X has a Uniform[0,3] distribution.
f. X has an Exponential(0.2) distribution.
g. X has a Normal(7,4) distribution.

13) Suppose X and Y are distributed as in the table below:

| | X=0 | X=2 | X=4 | p(y) |
|---|---|---|---|---|
| Y=0 | 0.5 | | | 0.7 |
| Y=1 | | 0.1 | | |
| p(x) | 0.6 | 0.2 | | |

a) Fill in the joint and marginal distributions for X and Y.
b) Find Cov(X,Y).
c) Are X and Y independent?
d) Find $P(X<3|Y=1)$.
e) Find Var(4X-2Y).

12) An airline knows from past experience that on an average only 90% of its passengers on a certain flight show up for their flight. So the airline routinely overbooks the flight.
Suppose for a flight with 180 seats, the airlines has given confirmed reservations to 194 passengers.
a. What is the exact distribution of X = number of passengers who show up for the flight?
b. What is the approximate probability that no passenger with a confirmed reservation has to be bumped from the flight?

13) A continuous random variable X taking values between 0 and 1 has the PDF
$$f(x) = kx^4 (1-x).$$

a. Find the value of k.
b. Find the CDF of X.
c. Find $P(X > 1/2)$.
d. Find the mean of X.
e. Is the median of X equal to the mean here? Why or why not?

14) The test scores on a GRE exam were normally distributed with a mean of 650 and variance 2500. Gabriel scored more than 95% of the examinees on this exam. Evaluate Gabriel's score on the exam.

15) A continuous random variable X has N(5,25) distribution. Identify which of the following random variables are also normally distributed and give their mean and variance. a. $-X$ ; b. $X^3$ ; c. $3X + 10$.

16) Suppose we use the following symbols to denote the respective distributions :
Normal = N; Exponential = E; Binomial = Bin; Poisson = Poi;
Geometric = G; Negative Binomial = NB.
For each of the following random variables, write the symbol that best describes the distribution of that random variable.
a. The number of free throws a player makes on 10 tries.
b. The number of e-mails received per day by the White House.
c. The amount of time a PC works before it fails for the first time.
d. The number of blind dates one must go on before meeting 3 nice people.

17) A discrete random variable X has mean 5 and standard deviation 5. Which of the following statements are correct?

18) a. X has a Poisson distribution.
b. $E(X^2) = 50$.
c. X has a Normal distribution.
d. $E(X - 5) = 0$.
e. $E(X - 5)^2 = 5$.

19) The claimed mean weight of Hostess Twinkies is 16 oz. Some students believe that the mean weight of Twinkies is less than what is claimed. They take a simple random sample of 12 Hostess Twinkies and find $\overline{x}$ to be 15.96 oz. Assume that weights of Hostess Twinkies are normally distributed with standard deviation $\sigma = 0.15$ oz.
a) State the null and alternative hypotheses.
b) Compute the test statistic.
c) Find the P-value.
d) State your conclusions in terms of the problem at the significance level $\alpha = 0.05$.
e) Suppose the student also wanted to compute a 95% confidence interval with a margin of error no larger than 0.05 oz. Did they take a large enough sample?
f) Suppose the mean weight of hostess Twinkies is in fact less than 16oz. Did the student make type I error or type II error? Explain.

20) It is known that the overall mean score $\mu$ for Test 2 for a population of students who have taken MA222 over the past five years is 78 and the standard deviation $\sigma$ is 8.

a) If you randomly choose a student who took MA 222 over the past five years, what is the probability that his or her score is larger than 80 points?
b) For a sample of 75 students taking MA222, what is the approximate distribution of the sample mean for this group?
c) Using part (b), what is the approximate probability that the sample mean for this group will be larger than 80 points?

Exam Sample(1 hr exam from 2000):

E1. For each of the following, circle **ALL** answers that correctly complete the statement. There is *at least one* correct answer for each problem.

a. (5 points) You know that a random variable X has mean 7 and variance 14. It is possible that

i. X ~ Normal
ii. X ~ Beta
iii. X ~ Exponential
iv. X ~ Uniform
v. X ~ Binomial

b. (5 points) Suppose that X is a normal random variable with variance 25. Then

i. there is about a 68% chance that X is within 5 units of its mean.
ii. there is about a 68% chance that X is within 25 units of its mean.
iii. there is about a 90% chance that X is within 10 units of its mean.
iv. there is about a 95% chance that X is within 10 units of its mean.
v. there is about a 99.7% chance that X is within 10 units of its mean.

c. (5 points) Suppose that X is a continuous random variable. Then it is <u>*always*</u> true that

i. the expectation of X is non-negative.
ii. the variance of X is non-negative.
iii. the PDF of X is a continuous function of X.
iv. the CDF of X is a continuous function of X.
v. $P(X<3) = P(X<=3)$

d. (5 points) If X is a random variable with $E(X) = 3$ and $Var(X) = 4$, then we can determine the value of

i. $E(3X+7)$
ii. $E(X^2+7)$
iii. $E(e^X+7)$
iv. $E(X^{-1})$
v. $E(E(X))$

E2. (5 points) You know *only* the mean and the standard deviation of a certain random variable X. You do not know anything else about its distribution. <u>*In one or two sentences*</u>, explain what you can say about the probability that X is bigger than a given constant *k*. You may give a mathematical formula if you like, but you do NOT need to do any calculations.

E3. Suppose X is a random variable with probability density function given by

$$f(x)=\begin{cases} kx^2(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. (4 points) Find the value of *k* that makes this function a probability density function.
b. (4 points) Find the CDF of X.
c. (4 points) Find P(1/2 < X < 3/4).
d. (8 points) Find the mean and variance of X. Also find E(C) where C = 2X+7.

E4. An airline knows that 80% of the passengers who reserve seats will show up for their flight. Because of this, on a particular flight with 500 seats, the airline accepts 600 reservations.

a. (2 points) What is the true distribution of X = the number of people who will show up for this flight?
b. (3 points) What is an approximate distribution for X that will help us avoid numerous calculations when computing probabilities?
c. (5 points) What is the probability that more than 500 people will show up for the flight (and hence the flight will be overbooked)?
d. (5 points) What is the probability that at least 460 but fewer than 490 people will board the flight?