REVIEW PROBLEMS

- 1. When Billy mows his grandfather's lawn, the amount of money he gets is a random variable determined as follows. First Billy flips a fair coin. If he gets heads, he tosses a fair dice once and gets X dollars, where X is the number showing on the die. If he gets tails on the coin flip, he rolls two fair dice and gets T dollars, where T is the total of the two numbers on the two dice.
 - (a) What is the probability that Billy gets paid exactly three dollars?
 - (b) You hear that Billy is disappointed because he was only paid three dollars the last time he mowed his grandfather's lawn. Given this information, what is the probability that Billy got heads on the coin flip?
- 2. Sally is hanging around a casino in Las Vegas. She has three \$1 chips. Sally decides to go bet at the roulette wheel according to the following strategy. She will keep making \$1 bets on black until she either wins a bet or until her chips are gone (whichever comes first). She has an 18/38 chance of winning each time she places a bet (since a Las Vegas roulette wheel has 38 equally likely pockets, 18 of which are black). She wins a chip when she wins a bet and loses a chip when she loses a bet.

Let X be the number of chips that Sally has when she quits betting. Calculate E(X).

- 3. The renowned surgeon, Dr. Bent Nagel, specializes in appendectomies and face lifts. Of the patients he sees, 20% have appendicitis, while the rest have bags under their eyes. Of those with appendicitis, 95% require surgery, while only 65% of those with baggy eyes go under his knife.
 - (a) If a patient visits him, what is the probability he will require surgery (regardless of the complaint)?
 - (b) If on the day you go to see him, he has just finished an operation, what is the probability that the patient had his appendix removed?
- 4. A deck of 52 cards is shuffled well. What is the probability that four aces are next to each other?
- 5. In 1693, the famous English diarist Samuel Pepys asked Isaac Newton whether it is more likely to get at least one six in six rolls of a fair die or to get at least two sixes in twelve rolls of a fair die. Determine which of the two has greater probability.

- 6. A subcommittee of three representatives is to be selected from five men and three women members of the House Ways and Means Committee. Find the probability that at least one man is on the subcommittee.
- 7. A pond contains 20 fish; 10 trout, 6 bass, and 4 carp. Four fish are randomly selected from the pond.
 - (a) What is the probability that three or more of the fish are trout?
 - (b) What is the probability that none of the fish are carp?
 - (c) What are the expected numbers of trout, bass and carp in the sample of four fish?
- 8. The Chicago Transit Authority has found that on a particular elevated train line, the amount of time between the stop at State and Madison and Howard Street is normally distributed with a mean of 25.5 minutes and a standard deviation of two minutes.
 - (a) What is the probability that a given train will take 30 minutes or more between these two stops?
 - (b) If 100 trains run each day along this route, on average, how many will take more than 30 minutes between these two stops?
 - (c) What is the probability that in the next ten trains, two or more take 30 minutes or more between stops?
- 9. Scores for Americans of age 20 to 34 on the Wechsler Adult Intelligence Test are normal with mean 100 and standard deviation 25. (Assume that fractional scores are possible.)
 - (a) What proportion of the scores is lower than 95?
 - (b) What proportion of the scores is between 104 and 127?
 - (c) Brian's score puts him at the 91st percentile, that is, 91% of the people have scores smaller than Brian's. What is his score?
- 10. Suppose that the number of typographical errors found on each page of a 280-page book has a Geometric distribution, with a mean of 2 errors per page.
 - (a) What is the probability that there are at least two errors on the last page?
 - (b) What is the probability that there are fewer than 500 errors in the whole book?
- 11. Flies enter through Grandma Jones' window as a Poisson process, with a rate of 10 per hour. That is, the expected number of flies that enter in an hour is $\lambda = 10$.
 - (a) What is the expected time until the first fly comes in the room?
 - (b) What is the probability that exactly 10 flies enter every hour, from 3 to 6pm?
- 12. Suppose that the average weight of rabbits is Normally distributed with a mean 12 pounds and a standard deviation of 1.8 pounds. What is the probability that in a random sample of 90 rabbits more than 15% of them will weigh over 14 pounds?

- 13. Suppose that S is a Poisson process with rate $\lambda = E(X_{(0,1)})$ and that S' is a different Poisson process, which independent of S, with rate λ' . Consider the sum of the two processes. That's the collection of all events, from both S and S'. Is it a Poisson process? (Hint: Use the definition of a Poisson process.)
- 14. Suppose X is a continuous random variable with density f(x) and distribution function F(x). Suppose $F(x) = 16x^4$ for $0 \le x \le 1/2$. Find F(1/4), f(1/4), F(2) and f(2).
- 15. Grandma Jones has a pretty regular sleep pattern: The amount of sleep she gets every night is uniformly distributed between 370 and 400 minutes.
 - (a) What is the probability that she slept longer than average 6.5 hours on 3 consecutive days?
 - (b) What is the probability that the average amount of sleep she got last month exceeded 6 hours and 28 minutes per night?