# Lecture 6 (week 4)

Tests of population variance

Two population variances

# Section7.3 Inference for variances

- Inference for population spread
- The *F* test for equality of variance
- The power of the two sample t-test

# Inference for population spread

It is possible to compare two population standard deviations $\sigma_1$ and $\sigma_2$ by comparing the standard deviations of two SRSs. However, these procedures are **not robust at all against deviations from normality**.

When $s_1^2$ and $s_2^2$ are sample variances from independent SRSs of sizes $n_1$ and $n_2$ drawn from normal populations, the $F$ statistic

$$F = s_1^2 / s_2^2$$

has the $F$ distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom when $H_0$: $\sigma_1 = \sigma_2$ is true.

The *F* distributions are right-skewed and cannot take negative values.

- The peak of the *F* density curve is near 1 when both population standard deviations are equal.
- Values of *F* far from 1 in either direction provide evidence against the hypothesis of equal standard deviations.

Table E in the back of the book gives critical *F*-values for upper *p* of 0.10, 0.05, 0.025, 0.01, and 0.001. We compare the *F* statistic calculated from our data set with these critical values for a one-side alternative; the *p*-value is doubled for a two-sided alternative.

$$F \quad has \quad \frac{Df_{numerator} : n_1 - 1}{Df_{denom} : n_2 - I}$$

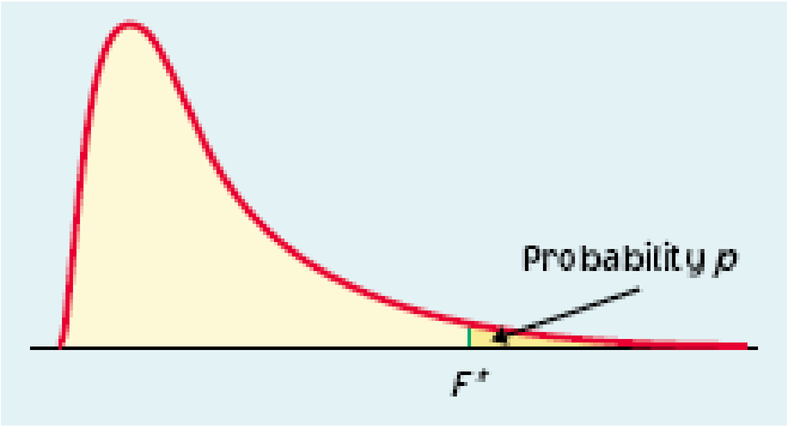


**Table E** *F* distribution critical values

$df_{num} = n_1 - 1$

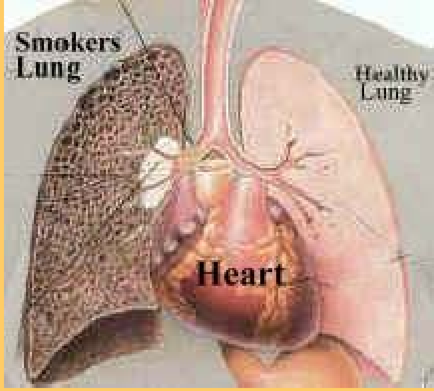
| Degrees of freedom in the denominator | $p$ | Degrees of freedom in the numerator 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.100 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 |
| | 0.050 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 |
| | 0.025 | 647.79 | 799.50 | 864.16 | 899.58 | 921.85 | 937.11 | 948.22 | 956.66 |
| | 0.010 | 4052.2 | 4999.5 | 5403.4 | 5624.6 | 5763.6 | 5859 | 5928.4 | 5981.1 |
| | 0.001 | 405284 | 500000 | 540379 | 562500 | 576405 | 585937 | 592873 | 598144 |
| 2 | 0.100 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 |
| | 0.050 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 |
| | 0.025 | 38.51 | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 |
| | 0.010 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 |
| | 0.001 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 |
| 3 | 0.100 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 |
| | 0.050 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 |
| | 0.025 | 17.44 | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 |
| | 0.010 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 |
| | 0.001 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 |
| 4 | 0.100 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 |
| | 0.050 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 |
| | 0.025 | 12.22 | 10.65 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 |
| | 0.010 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 |
| | 0.001 | 74.14 | 61.25 | 56.18 | 53.44 | 51.71 | 50.53 | 49.66 | 49.00 |
| 5 | 0.100 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 |
| | 0.050 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 |
| | 0.025 | 10.01 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 |
| | 0.010 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 |
| | 0.001 | 47.18 | 37.12 | 33.20 | 31.09 | 29.75 | 28.83 | 28.16 | 27.65 |
| 6 | 0.100 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 |
| | 0.050 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 |
| | 0.025 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 |
| | 0.010 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 |
| | 0.001 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 |

$p$

$F$

$df_{den} = n_2 - 1$

## Does parental smoking damage the lungs of children?




Forced vital capacity (FVC) was obtained for a sample of children not exposed to parental smoking and a group of children exposed to parental smoking.

| Parental smoking | FVC $\bar{x}$ | $s$ | $n$ |
|---|---|---|---|
| Yes | 75.5 | 9.3 | 30 |
| No | 88.2 | 15.1 | 30 |

$H_0$: $\sigma^2_{smoke} = \sigma^2_{no}$

$H_a$: $\sigma^2_{smoke} \neq \sigma^2_{no}$ (two sided)

$$F = \frac{\text{larger } s^2}{\text{smaller } s^2} = \frac{15.1^2}{9.3^2} \approx 2.64$$

The degrees of freedom are 29 and 29, which can be rounded to the closest values in Table E: 30 for the numerator and 25 for the denominator.

$2.54 < F(30,25) = 2.64 < 3.52$ ➔ $0.01 >$ 1-sided $p > 0.001$

➔ $0.02 >$ 2-sided $p > 0.002$

| F* | | Degrees of freedom (Df) in the numerator | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Proba | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 |
| | 0.100 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 | 1.87 | 1.77 | 1.72 | 1.66 |
| | 0.050 | 4.24 | 3.39 | 2.99 | 2.76 | 2.6 | 2.49 | 2.4 | 2.34 | 2.28 | 2.24 | 2.09 | 2.01 | 1.92 |
| 25 | 0.025 | 5.69 | 4.29 | 3.69 | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 | 2.61 | 2.41 | 2.3 | 2.18 |
| | 0.010 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.85 | 2.7 | 2.54 |
| | 0.001 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 | 4.06 | 3.79 | 3.52 |
| | 0.100 | 2.81 | 2.41 | 2.2 | 2.06 | 1.97 | 1.9 | 1.84 | 1.8 | 1.76 | 1.73 | 1.63 | 1.57 | 1.5 |
| | 0.050 | 4.03 | 3.18 | 2.79 | 2.56 | 2.4 | 2.29 | 2.2 | 2.13 | 2.07 | 2.03 | 1.87 | 1.78 | 1.69 |
| 50 | 0.025 | 5.34 | 3.97 | 3.39 | 3.05 | 2.83 | 2.67 | 2.55 | 2.46 | 2.38 | 2.32 | 2.11 | 1.99 | 1.87 |
| | 0.010 | 7.17 | 5.06 | 4.2 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.7 | 2.42 | 2.27 | 2.1 |
| | 0.001 | 12.22 | 7.96 | 6.34 | 5.46 | 4.9 | 4.51 | 4.22 | 4 | 3.82 | 3.67 | 3.2 | 2.95 | 2.68 |

# Power of the two-sample *t*-test

The power of the two-sample *t*-test against a specific alternative value of the difference in population means ($\mu_1 - \mu_2$) assuming a fixed significance level $\alpha$ is the probability that the test will reject the null hypothesis when the alternative is true.

The basic concept is similar to that for the one-sample *t*-test. The exact method involves the **noncentral *t* distribution.** Calculations are carried out with software.

You need information from a pilot study or previous research to calculate an expected power for your *t*-test and this allows you to plan your study smartly.

# Power calculations using a noncentral *t* distribution

For the pooled two-sample *t*-test, with parameters $\mu_1$, $\mu_2$, and the common standard deviation $\sigma$ we need to specify:

- An alternative that would be important to detect (i.e., a value for $\mu_1 - \mu_2$)
- The sample sizes, $n_1$ and $n_2$
- The Type I error for a fixed significance level, $\alpha$
- A guess for the standard deviation $\sigma$

We find the degrees of freedom df = $n_1 + n_2 - 2$ and the value of $t^*$ that will lead to rejection of $H_0$: $\mu_1 - \mu_2 = 0$

Then we calculate the **noncentrality parameter $\delta$**

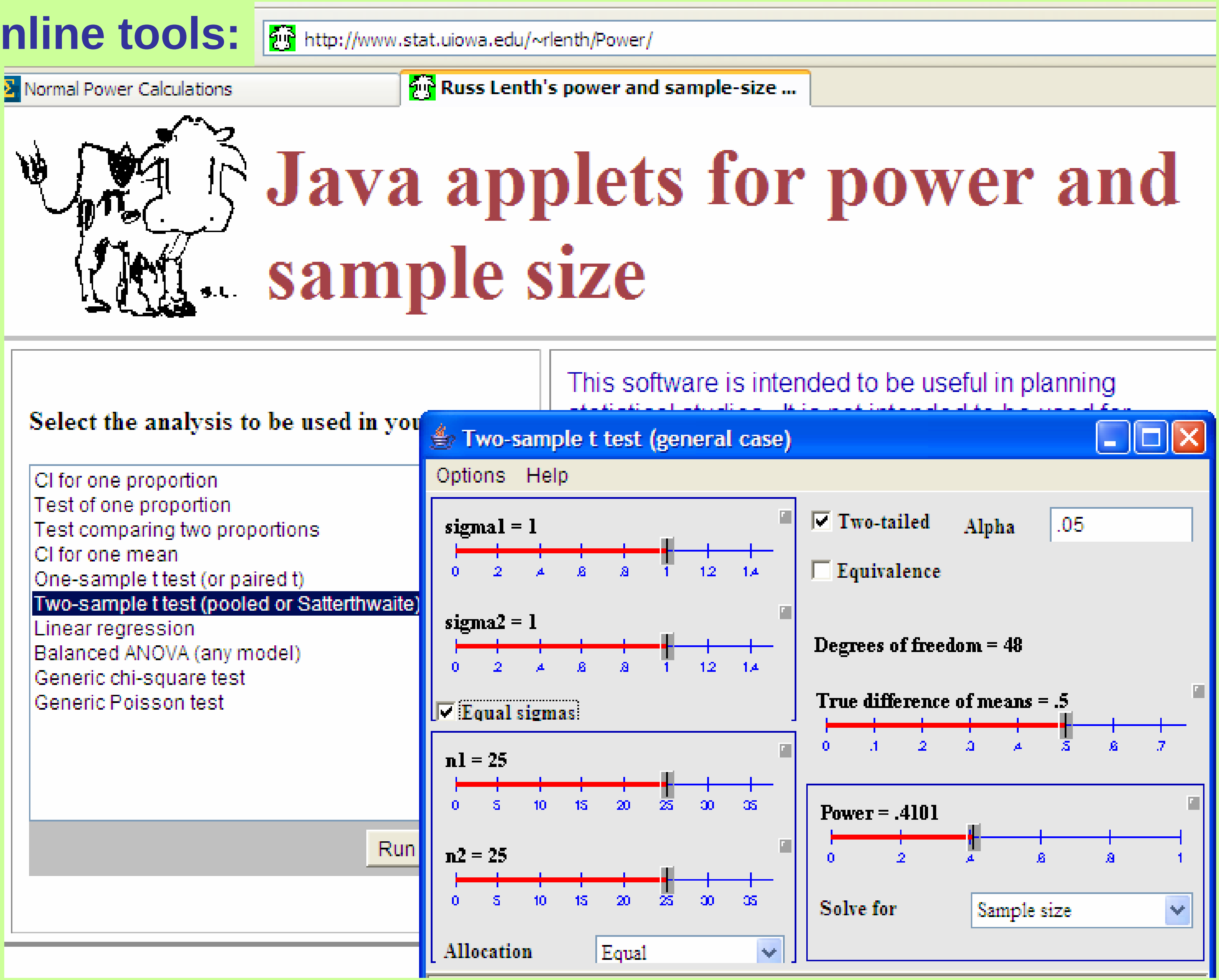
$$\delta = \frac{|\mu_1 - \mu_2|}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Finally, we find the power as the probability that a noncentral $t$ random variable with degrees of freedom df and noncentrality parameter $\delta$ will be greater than $t^*$:

- In R this is 1-pt*(*tstar*,* df*,* delta*)*. There are also several free online tools that calculate power.
- Without access to software, we can approximate the power as the probability that a standard normal random variable is greater than $t^* - \delta$, that is, $P(z > t^* - \delta)$, and use Table A.

For a ***test with unequal variances*** we can simply use the conservative degrees of freedom, but we need to guess both standard deviations and combine them for the guessed standard error: $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

# Online tools:



http://calculators.stat.ucla.edu/powercalc/

# Power Calculator

Choose a Model and Push a Button. Disclaimer.

| NORMAL | Power for a given Sample Size | Sample Size for a given Power |
|---|---|---|
| 1 Sample | ● | ● |
| 2 Sample, Equal Variances | ● | ● |
| 2 Sample, Unequal Variances | ● | ● |
| Lognormal | ● | ● |
| **EXPONENTIAL** | **Power for a given Sample Size** | **Sample Size for a given Power** |
| 1 Sample | ● | ● |
| 2 Sample | ● | ● |
| **BINO** | | |
| 1 Sample | | |
| 1 Sample Arcsir | | |
| 2 Sample Arcsir | | |
| 2 Sample Media | | |
| Fisher's Exact T | | |
| Proportion Resp | | |
| Case Control | | |
| **POIS** | | |
| 1 Sample | | |
| 2 Sample | | |
| **CORRELATION** | | |
| 1 Sample | | |

| **Enter a "?" for the item to be calculated.<br>Entering "?"s in positions 3 and 4 will calculate equal sample sizes for both groups.** | |
|---|---|
| **$\mu_1$**<br>The Mean of Population 1 | |
| **$\mu_2$**<br>The Mean of Population 2 | |
| **$N_1$**<br>The Sample Size from Population 1 | |
| **$N_2$**<br>The Sample Size from Population 2 | |
| **Sigma $_1$**<br>Standard Deviation of Group 1 | |
| **Sigma $_2$**<br>Standard Deviation of Group 2 | |
| **Significance Level**<br>The Significance Level of the test or Prob (reject null hypothesis ($H_0 : \mu_1 = \mu_2$) given it is true) | |
| **Power**<br>The Power desired for the test or Prob (reject $H_0$ given that $H_a$ is true) | |
| **Number of Sides**<br>Specifies Alternative Hypothesis.<br>One sided and $\mu_1 > \mu_2$ => $H_1 : \mu_1 > \mu_2$<br>One sided and $\mu_1 < \mu_2$ => $H_1 : \mu_1 < \mu_2$<br>Two sided => $H_1 : \mu_1$ not equal $\mu_2$ | ◉ 1 Side<br>○ 2 Sides |
| Calculate | |

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