Problem 1 solution:

In this problem, the independent variable is t, the dependent variable is y. Writing the equation in the form

$$\frac{dy}{dt} = ty + \sin^2 t\,,$$

we conclude that $f(t,y) = ty + \sin^2 t$, $\partial f(t,y)/\partial y = t$. Both functions, f and $\partial f/\partial y$, are continuous on the whole ty-plane. So, Theorem 1 applies for any initial condition, in particular, for $y(\pi) = 5$.

Problem 2 solution: not given

Problem 3 solution:

We have

$$\begin{aligned}
\frac{3v\,dv}{1-4v^2} = \frac{dx}{x} \qquad &\Rightarrow \qquad \int \frac{3v\,dv}{1-4v^2} = \int \frac{dx}{x} \\
\Rightarrow \qquad & -\frac{3}{8}\int \frac{du}{u} = \int \frac{dx}{x} \qquad (u = 1-4v^2,\ du = -8v\,dv) \\
\Rightarrow \qquad & -\frac{3}{8}\ln\left|1-4v^2\right| = \ln|x| + C_1 \\
\Rightarrow \qquad & 1-4v^2 = \pm\exp\left[-\frac{8}{3}\ln|x| + C_1\right] = Cx^{-8/3},
\end{aligned}$$

where $C = \pm e^{C_1}$ is any nonzero constant. Separating variables, we lost constant solutions satisfying

$$1 - 4v^2 = 0 \qquad \Rightarrow \qquad v = \pm\frac{1}{2}\,,$$

which can be included in the above formula by letting $C = 0$. Thus,

$$v = \pm\frac{\sqrt{1 - Cx^{-8/3}}}{2}\,, \quad C \text{ arbitrary},$$

is a general solution to the given equation.

Problem 4 solution: In the order presented they are:

Linear

Neither

Neither

Linear

Both

Linear

Problem 5 solution:

Since $\mu(x) = \exp\left(\int 4dx\right) = e^{4x}$, we have

$$\frac{d}{dx}\left(e^{4x}y\right) = e^{4x}e^{-x} = e^{3x}$$

$$\Rightarrow \qquad y = e^{-4x}\int e^{3x}dx = \frac{e^{-x}}{3} + Ce^{-4x}.$$

Substituting the initial condition, $y = 4/3$ at $x = 0$, yields

$$\frac{4}{3} = \frac{1}{3} + C \qquad \Rightarrow \qquad C = 1,$$

and so $y = e^{-x}/3 + e^{-4x}$ is the solution to the given initial value problem.