Section 6.1 problems:

Problem 10 solution:

Linearly independent; $-2\tan^3 x - \sin x \cos x - \sin^2 x \tan x - 2\tan x$

Problem 14 solution:

Linearly independent; $(x+2)e^x$

Problem 18 solution:

$c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

Problem 20 solution:

(a) $c_1 + c_2 x + c_3 x^3 + x^2$

(b) $2 - x^3 + x^2$

Problem 32 solution: not provided.

Section 4.2 problems:

Problem 10 solution:

Solving the auxiliary equation, $4r^2 - 4r + 1 = (2r-1)^2 = 0$, we conclude that $r = 1/2$ is its double root. Therefore, a general solution to the given differential equation is

$$y(t) = c_1 e^{t/2} + c_2 t e^{t/2}.$$

Problem 16 solution:

The auxiliary equation for this problem, $r^2 - 4r + 3 = 0$, has roots $r = 1, 3$. Therefore, a general solution is given by

$$y(t) = c_1 e^t + c_2 e^{3t} \qquad \Rightarrow \qquad y'(t) = c_1 e^t + 3c_2 e^{3t}.$$

Substitution of $y(t)$ and $y'(t)$ into the initial conditions yields the system

$$\begin{aligned} y(0) &= c_1 + c_2 = 1 \\ y'(0) &= c_1 + 3c_2 = 1/3 \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} c_1 &= 4/3 \\ c_2 &= -1/3. \end{aligned}$$

Thus, the solution satisfying the given initial conditions is

$$y(t) = \frac{4}{3}e^t - \frac{1}{3}e^{3t}.$$

Problem 30 solution:

These functions are linearly independent, because the equality $y_1(t) \equiv c y_2(t)$ would imply that

$$t^2 \cos(\ln t) \equiv c t^2 \sin(\ln t) \qquad \Rightarrow \qquad \cot(\ln t) \equiv c$$

on $(0, 1)$, which is false.

Problem 44 solution:

First we find a general solution to the equation $y''' - 2y'' - y' + 2y = 0$. Its characteristic equation, $r^3 - 2r^2 - r + 2 = 0$, has roots $r = 2$, 1, and -1, and so a general solution is given by

$$y(t) = c_1 e^{2t} + c_2 e^t + c_3 e^{-t}.$$

Differentiating $y(t)$ twice yields

$$y'(t) = 2c_1 e^{2t} + c_2 e^t - c_3 e^{-t}, \qquad y''(t) = 4c_1 e^{2t} + c_2 e^t + c_3 e^{-t}.$$

Now we substitute y, y', and y'' into the initial conditions and find c_1, c_2, and c_3.

$$\begin{array}{rcl} y(0) = c_1 + c_2 + c_3 & = & 2 \\ y'(0) = 2c_1 + c_2 - c_3 & = & 3 \\ y''(0) = 4c_1 + c_2 + c_3 & = & 5 \end{array} \quad \Rightarrow \quad \begin{array}{l} c_1 = 1 \\ c_2 = 1 \\ c_3 = 0. \end{array}$$

Therefore, the solution to the given initial value problem is

$$y(t) = e^{2t} + e^t.$$