**Review Final MA221 Summer 2009**

I. Solve:

(a)
$$y'' - y' + y = 0$$

(b)
$$x^2y'' - 5xy' + 25y = 0$$

II. (a) Find the general solution of

$$y'' - 4y' + 3y = 2e^{3x} + \sin x$$

by using the method of undetermined coefficients.

(b) Assuming that $y_1 = e^x$ and $y_2 = e^x \ln x$ are linearly independent, find the general solution of:

$$xy'' + (1 - 2x)y' + (x - 1)y = xe^x$$

III. Solve

(a)
$$\frac{dy}{dx} + 2y = y^2$$

(b)
$$2xydx + (3x^2 + 3)dy = 0$$

IV. Find the following:

(a)
$$\mathcal{L}\{f(t)\}(s) \quad \text{where } f(t) = \begin{cases} 2 & 0 \le t \le 5 \\ 0 & t > 5 \end{cases}$$

(b)
$$\mathcal{L}\{te^{2t} \sin 3t\}(s)$$

V. Find:

(a)
$$\mathcal{L}^{-1}\{\frac{7s^2 - 11s - 5}{(s-1)^2(s+2)}\}(t)$$

(b) Solve, using Laplace transforms

$$y'' + 4y' + 20y = 0 \qquad y(0) = 2 \qquad y'(0) = 1$$

VI. (a) Find the $1^{st}$ six non-zero terms of a power series expansion of the solution of

(a) $(x^2 + 4)y'' + (x - 1)y' + 3y = 0 \qquad y(2) = 1 \qquad y'(2) = 3$

(b) Find a general expression for the solution of the following equation in terms of power series about $x = 0$.

$$(1 + x^2)y'' + 3y = 0$$

VII. Find the first four non-zero terms of the Fourier cosine series of

$$f(x) = \begin{cases} 0 & \text{if } 0 < x < 2 \\ 2 & \text{if } 2 < x < 4 \end{cases}$$

VIII. Find all eigenvalues and eigenfunctions of

$$y'' - 4y' + \lambda y = 0 \qquad y(0) = 0 \qquad y(1) = 0$$

IX. Solve

$$\begin{aligned} &u_{tt} = 4u_{xx} && 0 < x < 3, t > 0 \\ &u_x(0,t) = u_x(3,t) = 0 && t > 0 \\ &u(x,0) = 5\cos\frac{7\pi x}{3} - 11\cos\frac{13\pi x}{3} && 0 < x < 3 \\ &u_t(x,0) = 2\cos\frac{5\pi x}{3} - 3\cos\frac{11\pi x}{3} && 0 < x < 3 \end{aligned}$$

Assume eigenvalues are not positive.

X. Find a formal solution to the given PDE:

$$\begin{aligned} &u_{xx} + u_{yy} = 0, && 0 < x < \pi, 0 < y < \pi \\ &u(0,y) = u(\pi,y) = 0, && 0 < y < \pi \\ &u(x,0) = \sin x + \sin 4x, && 0 < x < \pi \\ &u(x,\pi) = 0, && 0 < x < \pi \end{aligned}$$

XI. Find a formal solution to the given PDE:

$$\frac{\partial u}{\partial t} = 7\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, 0 < t$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0, \qquad 0 < t$$
$$u(x,0) = 1 - \sin x, \qquad 0 < x < \pi$$

XII. Use a method learned in this class to find at least the first nonzero terms in the series expansion about $x = 0$ for the solution of the equation:

$$xy'' + y' - 4y = 0, \quad x > 0$$

XIII. Determine an inverse Laplace transformation of the function:

$$\frac{e^{-s}(3s^2 - s + 2)}{(s-1)(s^2+1)}$$