Homework 1 – Due May 20

1. Determine whether Theorem 1 (Existence and Uniqueness for first-order initial value problem) is applicable and whether the following ODE admits a unique solution or not:

$$\frac{dy}{d\theta} - \theta y = (\sin\theta)^2 \qquad y(\pi) = 5$$

2. Determine whether Theorem 1 (Existence and Uniqueness for first-order initial value problem) is applicable and the following ODE admits a unique solution:

$$x\frac{dx}{dt} + 4t = 0 \qquad x(2) = -\pi$$

3. Solve the equation:

$$x\frac{dv}{dx} = \frac{1 - 4v^2}{3v}$$

4. Determine whether the given equations are separable, linear, neither, or both:

$$x^2\frac{dy}{dx} + \cos x = y$$
$$\frac{dx}{dt} + xt = e^x$$
$$x\frac{dx}{dt} + t^2x = \sin t$$
$$3t = e^t\frac{dy}{dt} + y\ln t$$
$$(t^2 + 1)\frac{dy}{dt} = yt - y$$
$$3r = \frac{dr}{d\theta} - \theta^3$$

5. Solve the initial value problem:

$$\frac{dy}{dx} + 4y - e^{-x} = 0 \qquad y(0) = \frac{4}{3}$$