Homework 3

Ma623 Stochastic Processes

due Tuesday March 24 2009

For this assignment please do the following problems from the pages 229-237 (ch 5) of your textbook: page 229 ex. 6, page 230 ex. 8 (alternating renewal process), page 230 ex. 2, page 232 ex. 12.

In addition do the following exercises:

(1) Consider a single-server bank (a bank with only one office) in which potential customers arrive at a Poisson rate λ. However, an arrival only enters the bank if the server is free when he or she arrives. Let G denote the service distribution.

   (a) At what rate do customers enter the bank?
   (b) What fraction of potential customers enter the bank?
   (c) What fraction of the time is the server busy?

   Simulation part Now let us try and use simulation to solve this problem. Assume that $\lambda = 2$ customers per minute, and that $G = Uniform[0, 1]$. Use software to generate the Poisson process of the arrivals and the times of the service (the blackout periods). Now calculate the new arrival process.

   (d) Using the elementary renewal theorem you were able to calculate in part (a) the average rate of the new process when t is large. Now use simulation to do the same thing. Use $t = 10,000$ minutes and as many repetitions as you think necessary.
   (e) Again using the simulation answer parts (b) and (c). Use the same value for t as above.
   (f) Calculate using the theory the answers for the particular case considered in the simulation for your specific values of λ and G. Then record and give the order of difference between the theoretical values and the simulation.

(2) Trucks arrive at a UPS station according to a renewal process with $U(0,1)$ (in hours) interarrival times. All packages waiting at that station are instantly loaded as soon as a truck arrives. Packages arrive at the UPS station according to a Poisson process with rate 4/hour. Calculate:

$$\lim_{t\to\infty} \mathbf{P}\{\textbf{NO} \text{ packages at the station at time } t\}$$

(3) A fair six sided die has sides: $10, 15, 25, 40, 45, 75$. Let S_n be the sum of the first n rolls and $N(t)$ the number of times the die was rolled before reaching the total t.

Calculate:

(a) $\mathbf{P}(S_n = 2,678,495 \text{ for some } n)$

(b) The 95^{th} percentile of $N(2,678,495)$

(4) A critical component of the next space shuttle to Europa (the Jupiter satellite) has an operating lifetime that is exponentially distributed with mean 1 year (ship time). As soon as a component fails it is replaced by a new one having statistically identical properties. It is known that the one-way travel to Europa lasts exactly 2 years. What is the smallest number of components that the shuttle should stock if it wishes that probability of having an inoperable unit caused by failures exceeding the spare inventory throughout the time of the **voyage** to be less than 0.02?

(5) A certain type network switch has two states: 0=OFF and 1=OPERATIONAL. In state 0 there is no data flowing through the switch. The component remains in state 0 a random amount of time that is exponentially distributed with rate α and then moves to state 1. The time in state 1 is exponentially distributed with rate β, after which the switch returns to state 0.

The causes of the existence of state 0 may be malevolent hackers, mechanical, and electrical failure etc.

One of the CS internet connections (call it CSIC) has two of these switches (denoted A and B), connected in parallel. That means that in order for the CSIC to function at least one of the switches A or B must be operating. The two switches have the following parameters:

Switch	Operating failure rate	Repair rate
A	$\beta_A = 0.01$	$\alpha_A = 0.1$
B	$\beta_B = 0.002$	$\alpha_B = 0.01$

Assume that the two switches operate independently of each other.

(a) In the long run what fraction of time is the internet connection (CSIC) down?

(b) Once the CSIC is down what is the mean duration time prior to returning to operation?

(c) Define a cycle as the time between the instant that the CSIC enters the down state and the next such instant. Find the mean duration of such a cycle.

(d) What is the mean duration between failures?

Please note that replacing simulation results for theoretical answers is possible. It will earn you credit but only partial credit.