Homework 4

Ma623 Stochastic Processes

due Tuesday April 7 2009

For this assignment please solve the following problems:

(1) Consider a nonsymmetric random walk with history on $\mathbb{Z}$, where increments are ± 1 and which has the property:

$$\mathbf{P}\{X_n = 1 | X_1, X_2, \dots, X_{n-1}\} \geq \alpha > 0, \forall n$$

Prove that:

(a) $\mathbf{P}\{X_n = 1 \text{ for some } n\} = 1$

(b) $\mathbf{P}\{X_n = 1 \text{ infinitely often}\} = 1$

(2) Let a 2×2 matrix be:

$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$$

Calculate the n-th power of this matrix: P^n.

(3) Consider a random walk $\{S_n\}_n$ on integers $\mathbb{Z}$ such that the increments have probability $\mathbf{P}\{X_i = 1\} = p = 1 - \mathbf{P}\{X_i = -1\}$ and let $1 - p = q$.

(a) Determine the probability:

$$P_{00}^n = \mathbf{P}\{X_n = 0 | X_0 = 0\}$$

(Note: this we actually calculated in class).

(b) Find the generating function of the sequence $u_n = P_{00}^n$, i.e.:

$$G(x) = \sum_{n=0}^{\infty} u_n x^n$$