MA222 – HW3

Ex. 50 / Pg. 114

$X \sim Bin(25,.25)$

a. $P(X \le 6) = B(6;25,.25) = .561$

b. $P(X = 6) = b(6;25,.25) = .183$

c. $P(X \bullet 6) = 1 – P(X \bullet 5) = 1 – B(5;25,.25) = .622$

d. $P(X > 6) = 1 – P(X \bullet 6) = 1 – .561 = .439$

Ex. 51 / Pg. 114

a. $E(X) = np = 25(.25) = 6.25$

b. $Var(X) = np(1–p) = 25(.25)(.75) = 4.6875$, so $SD(X) = 2.165$

c. $P(X > 6.25 + 2(2.165)) = P(X > 10.58) = 1 – P(X \bullet 10.58) = 1 – P(X \bullet 10) = 1 – B(10;25,.25) = .030$

Ex. 54 / Pg. 114

$X \sim Bin(10, .60)$

a) $P(X \ge 6) = 1 – P(X \le 5) = 1 - B(5;20,.60) = 1 - .367 = .633$

b) $E(X) = np = (10)(.6) = 6$; $V(X) = np(1 – p) = (10)(.6)(.4) = 2.4$;
$\sigma_x = 1.55$
$E(X) \pm \sigma_x = (4.45, 7.55)$.
We desire $P(5 \le X \le 7) = P(X \le 7) – P(X \le 4) = .833 - .166 = .667$

c) $P(3 \le X \le 7) = P(X \le 7) – P(X \le 2) = .833 - .012 = .821$

Ex. 69 / Pg. 120

2. $X \sim h(x; 6, 12, 7)$

a. $P(X=5) = \dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \dfrac{105}{924} = .114$

b. $P(X \bullet 4) = 1 – P(X \bullet 5) = 1 – [P(X=5) + P(X=6)] =$

$$1-\left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}+\frac{\binom{7}{6}}{\binom{12}{6}}\right]=1-\frac{105+7}{924}=1-.121=.879$$

c. E(X) = $\left(\frac{6 \cdot 7}{12}\right) = 3.5$; $\sigma = \sqrt{\left(\frac{6}{11}\right)(6)\left(\frac{7}{12}\right)\left(\frac{5}{12}\right)} = \sqrt{.795} = .892$ P(X > 3.5 + .892) = P(X > 4.392) = P(X• 5) = .121 (see part b)

d. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large: h(x;15,40,400) approaches b(x;15,.10). So P(X• 5) • B(5; 15, .10) from the binomial tables = .998

Ex. 75 / Pg. 121

a. With S = a female child and F = a male child, let X = the number of F's before the 2^{nd} S. Then P(X = x) = nb(x;2, .5)

b. P(exactly 4 children) = P(exactly 2 males)
= nb(2;2,.5) = (3)(.0625) = .188

c. P(at most 4 children) = P(X ≤ 2)
$= \sum_{x=0}^{2} nb(x;2,.5)$ = .25+2(.25)(.5) + 3(.0625) = .688

d. E(X) = $\frac{(2)(.5)}{.5} = 2$, so the expected number of children = E(X + 2)
= E(X) + 2 = 4

Ex. 82 / Pg. 125

a. P(X = 1) = F(1;.2) – F(0;.2) = .982 - .819 = .163

b. P(X ≥ 2) = 1 – P(X ≤ 1) = 1 – F(1;.2) = 1 - .982 = .018

c. P(1^{st} doesn't $\cap$ 2^{nd} doesn't) = P(1^{st} doesn't) · P(2^{nd} doesn't)
= (.819)(.819) = .671

Ex. 100 / Pg. 127 (BONUS)

a. P(X ≥ 5) = 1 - B(4;25,.05) = .007

b. P(X ≥ 5) = 1 - B(4;25,.10) = .098

c. P(X ≥ 5) = 1 - B(4;25,.20) = .579

d. All would decrease, which is bad if the % defective is large and good if the % is small.