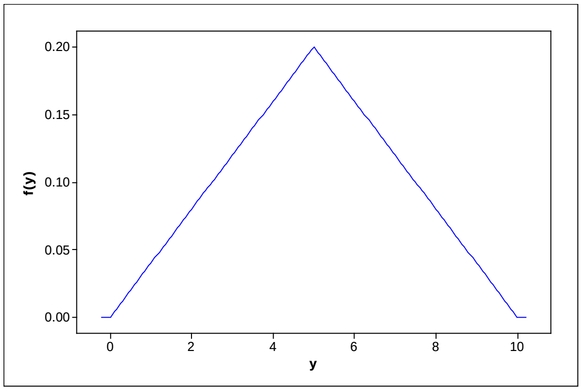
MA222 – HW4

Ex. 8 / Pg. 135

a.



b. $\int_{-\infty}^{\infty} f(y)dy = \int_0^5 \frac{1}{25}ydy + \int_5^{10} (\frac{2}{5} - \frac{1}{25}y)dy = \left.\frac{y^2}{50}\right]_0^5 + \left.\left(\frac{2}{5}y - \frac{1}{50}y^2\right)\right]_5^{10}$

$= \frac{1}{2} + \left[(4-2) - (2 - \frac{1}{2})\right] = \frac{1}{2} + \frac{1}{2} = 1$

c. P(Y ≤ 3) = $\int_0^3 \frac{1}{25}ydy = \left.\frac{y^2}{50}\right]_0^3 = \frac{9}{50} \approx .18$

d. P(Y ≤ 8) = $= \int_0^5 \frac{1}{25}ydy + \int_5^8 (\frac{2}{5} - \frac{1}{25}y)dy = \frac{23}{25} \approx .92$

e. P(3 ≤ Y ≤ 8) = P(Y ≤ 8) – P(Y < 3) = $\frac{46}{50} - \frac{9}{50} = \frac{37}{50} = .74$

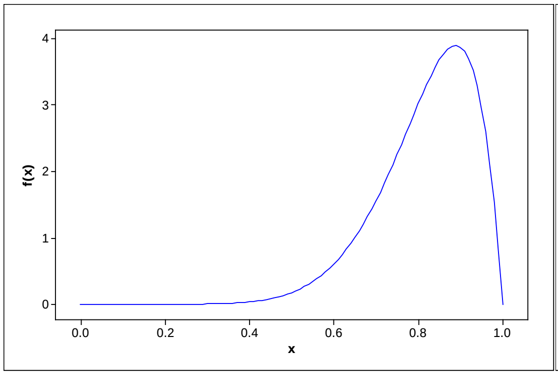
f. P(Y < 2 or Y > 6) = $= \int_0^2 \frac{1}{25}ydy + \int_6^{10} (\frac{2}{5} - \frac{1}{25}y)dy = \frac{2}{5} = .4$

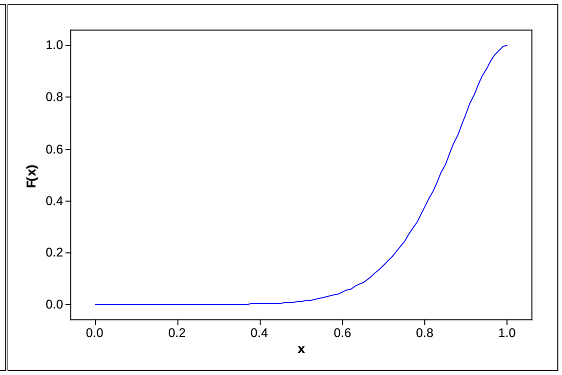
Ex. 15 / Pg. 143

a. F(x) = 0 for x ≤ 0, = 1 for x ≥ 1, and for 0 < x < 1,

$$F(x) = \int_{-\infty}^{x} f(y)dy = \int_0^x 90y^8(1-y)dy = 90\int_0^x (y^8 - y^9)dy$$

$$90\left(\tfrac{1}{9}y^9 - \tfrac{1}{10}y^{10}\right)\Big]_0^x = 10x^9 - 9x^{10}$$





b. $F(.5) = 10(.5)^9 - 9(.5)^{10} \approx .0107$

c. $P(.25 \le X \le .5) = F(.5) - F(.25) \approx .0107 - [10(.25)^9 - 9(.25)^{10}]$
$\approx .0107 - .0000 \approx .0107$

d. The 75th percentile is the value of x for which F(x) = .75
$\Rightarrow .75 = 10(x)^9 - 9(x)^{10}$ $\Rightarrow x \approx .9036$

e. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^1 x \cdot 90x^8(1-x)dx = 90\int_0^1 x^9(1-x)dx$

$= 9x^{10} - \frac{90}{11}x^{11}\Big]_0^1 = \frac{9}{11} \approx .8182$

$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx = \int_0^1 x^2 \cdot 90x^8(1-x)dx = 90\int_0^1 x^{10}(1-x)dx$

$= \frac{90}{11}x^{11} - \frac{90}{12}x^{12}\Big]_0^1 \approx .6818$

$V(X) \approx .6818 - (.8182)^2 = .0124$, $\sigma_x = .11134$.

f. $\mu \pm \sigma = (.7068, .9295)$. Thus, $P(\mu - \sigma \le X \le \mu + \sigma) = F(.9295) - F(.7068) = .8465 - .1602 = .6863$, so the probability X is *more* than 1 sd from its mean equals 1–.6863 = 3137.

<u>Ex. 17 / Pg. 143</u>

a. $F(X) = \frac{x-A}{B-A} = p \Rightarrow$ x = (100p)th percentile = A + (B – A)p

b. $E(X) = \int_A^B x \cdot \frac{1}{B-A}dx = \frac{1}{B-A} \cdot \frac{x^2}{2}\Big]_A^B = \frac{1}{2} \cdot \frac{1}{B-A} \cdot (B^2 - A^2) = \frac{A+B}{2}$

$E(X^2) = \frac{1}{3} \cdot \frac{1}{B-A} \cdot (B^3 - A^3) = \frac{A^2 + AB + B^2}{3}$

$V(X) = \left(\frac{A^2 + AB + B^2}{3}\right) - \left(\frac{(A+B)}{2}\right)^2 = \frac{(B-A)^2}{12}, \ \sigma_x = \frac{(B-A)}{\sqrt{12}}$

$$E(X^n) = \int_A^B x^n \cdot \frac{1}{B-A}\,dx = \frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}$$

Ex. 21 / Pg. 144

E(area) = E(πR^2) = $\int_{-\infty}^{\infty} \pi r^2 f(r)dr = \int_9^{11} \pi r^2 \left(\frac{3}{4}\right)\left(1-(10-r)^2\right)dr = \frac{501}{5}\pi = 314.79$

Ex. 22 / Pg. 144

a. For $1 \le x \le 2$, F(x) = $\int_1^x 2\left(1-\frac{1}{y^2}\right)dy = 2\left(y+\frac{1}{y}\right)\Big]_1^x = 2\left(x+\frac{1}{x}\right)-4$, so

$$F(x) = \begin{cases} 0 & x < 1 \\ 2\left(x+\frac{1}{x}\right)-4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b. $2\left(x_p + \frac{1}{x_p}\right) - 4 = p \Rightarrow 2x_p^2 - (4-p)x_p + 2 = 0 \Rightarrow x_p = \frac{1}{4}[4 + p + \sqrt{p^2 + 8p}]$ To find $\tilde{\mu}$, set p = .5 $\Rightarrow \tilde{\mu}$ = 1.64

c. E(X) = $\int_1^2 x \cdot 2\left(1-\frac{1}{x^2}\right)dx = 2\int_1^2 \left(x - \frac{1}{x}\right)dx = 2\left(\frac{x^2}{2} - \ln(x)\right)\Big]_1^2 = 1.614$

E(X^2) = $2\int_1^2 \left(x^2 - 1\right)dx = 2\left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3} \Rightarrow$ Var(X) = .0626

d. Amount left = max(1.5 – X, 0), so

E(amount left) = $\int_1^2 \max(1.5-x,0)\, f(x)dx = 2\int_1^{1.5} (1.5-x)\left(1-\frac{1}{x^2}\right)dx = .061$