Midterm Exam 2, Spring 2008 (Take Home Exam)

due Monday 28 June, 2010 by class time (1:30pm)

Name:

- There are 5 problems, each worth 20 points for a total of 100.
- Be very specific with your random variables and your definitions. Show-case your work.
- Please write neatly and clearly
- You may attach as many pages as you think are necessary. Please use one full page for each problem in this exam.

For instructor's use only

Problem	Points	Score
1	15	
2	20	
3	20	
4	30	
5	15	
Total	100	

1. Show that the correlation is invariant to linear transformations. That is for any two random variables X and Y with joint pdf $f(x, y)$ prove that the correlation between X and Y is the same as the correlation between $aX + b$ and $cY + d$ for any a, b, c, d real numbers.

2. Give an example of two random variables X and Y which are uncorrelated ($\rho_{X,Y} = 0$) but not independent. You may give an example of either discrete or continuous r.v. but in either case do not forget to write clearly the joint distribution.

3. A point is selected at random from the unit disk

$$\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$$

Let X be the x coordinate and Y be the y coordinate of the point chosen. Determine if X and Y are independent random variables. (A non-mathematical proof will earn 0 points).

4. Let the joint pdf of two variables X and Y be:

$$f(x, y) = \begin{cases} cx(1-x) & \text{if } 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the value of c which makes $f(x, y)$ a true probability density
   (b) Calculate the marginal distributions of x and Y
   (c) Calculate the conditional distribution of $Y|X = x$
   (d) Determine if X and Y are independent
   (e) Calculate the Covariance and correlation of X and Y
   (f) Calculate the conditional probability:

$$P(1/2 \leq Y \leq 2/3 \mid X = 1/2)$$

5. A bar of length L is broken into three pieces at two random points. What is the probability that the length of at least one piece is less than $L/20$?