FE 610. Assignment 1

due Tuesday September 10, 2012 at the beginning of the class (6:15pm).

Please note that due to the Labor day this assignment has two weeks to complete. The grade for this assignment is double the regular assignments (respectively 20 points).

1. Calculate the Total Variation (on $\mathbb{R}$) of the function:

$$f(x) = \begin{cases} 1 & \text{for } x < 0, \\ \cos x & \text{for } x \in [0, 2\pi], \\ 3 & \text{for } x > 2\pi, \end{cases}$$

2. Consider the functions $f_1, f_2 : (0, \infty) \to \mathbb{R}$, $f_1(x) = \sqrt[3]{x} \sin\left(\frac{\pi}{x}\right)$ and $f_2 = x^2 \sin\left(\frac{\pi}{x}\right)$. Investigate if these functions have bounded variation on the interval $[0, 1]$ and if they do calculate (or approximate) this value.

3. Find the global max and min of the following functions:

   a) $\log(x^3 - 27)$ for $x > 3$

   b) $\sqrt[3]{x^2}(x-1)^4$ for $x \in (0, 1)$

4. Let $x_0 \in \mathbb{R}$ and $f \in \mathcal{C}^2(\mathbb{R})$ (twice derivable functions with second

derivative continuous). Let

$$M_0 = \sup_{x>x_0} |f(x)|$$
$$M_1 = \sup_{x>x_0} |f'(x)|$$
$$M_2 = \sup_{x>x_0} |f''(x)|$$

where f' and f'' denote first respectively second derivative of the function f, and suppose all $M_0, M_1 M_2$ are finite. Show that

$$M_1^2 \leq 4 M_0 M_2$$

Hint: Consider the Taylor formula with reminder term (2nd order approximation). Then, recall the sign of a quadratic expression.

5. Let $f \in \mathcal{C}^2((0,\infty))$ with the first derivative f' bounded on $(0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$. Show that

$$\lim_{x\to\infty} f'(x) = 0$$

Hint: Use the previous problem. Note that x_0 can be any arbitrary number.

Next problems are probability review. You should do these and any others from areas you feel you are not conformable with.

6. Gottfried Wilhelm Leibnitz (1646-1716), German mathematician, philosopher, statesman and one of the supreme intellects of the seventeenth century believed that in a throw of a pair of fair dice (labeled 1 to 6) the probability of obtaining the sum 11 is the same as obtaining the sum 12. Do you agree? Explain.

7. 6 faculty members in the Mathematical Sciences department have declared that their computers are no longer keeping up with the current day technology. The department gave them a choice and thus four members selected desktop computers and two requested laptop computers as replacement. However, soon after the department found out that funding is only available for two computers (regardless of type). Thus they decided to use random drawing to decide on the replacements.

a) The Department does not care who is going to receive the computers. It is only interested in the type of computers to be bought. List the outcomes in the sample space from the department perspective.

b) The faculty members on the other hand are very much interested in who will receive what. Use your own notation and write the sample space from the faculty members' perspective.

c) List the outcomes in the event "Both computers chosen were laptops". Use both sample spaces from a) and b)

d) Professor Florescu is one of the members of the department who selected a replacement laptop. List all the outcomes in the event "professor Florescu will receive a replacement laptop". Can you use both sample spaces to write this event? Why? /Why not?

e) Calculate the probability of the event in d)

8. In *USA Today* (Sep. 5, 1996) the results of a survey involving the use of sleepwear while traveling were listed as follows:

	Male	**Female**	
Underwear	0.220	0.024	0.244
Nightgown	0.002	0.180	0.182
Nothing	0.160	0.018	0.178
Pajamas	0.102	0.073	0.175
T-shirt	0.046	0.088	0.134
Other	0.084	0.003	0.087

(a) What is the probability that a traveler is a female who sleeps in the nude?

(b) What is the probability that the traveler is a male?

(c) Assuming that the traveler is a male, what is the probability that he sleeps in the pajamas?

(d) What is the probability that a traveler is male if he/she sleeps in pajamas or a T-shirt?

9. Let two random variables X and Y have joint distribution

$$f(x,y) = \frac{21}{4}x^2 y, \text{ if } x^2 \leq y \leq 1$$

(a) Calculate the marginal densities of X and Y

(b) Calculate $\mathbf{E}[X]$ and $\mathbf{E}[Y]$

(c) Calculate the conditional densities of $X|Y$ and $Y|X$

(d) Calculate the conditional expectations $\mathbf{E}[X|Y]$ and $\mathbf{E}[Y|X]$

10. Problem 3 on page 118 in the Neftci textbook.