

# Adaptive Distributed Estimation of Signal Power from One-Bit Quantized Data

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**Abstract**—We examine distributed estimation of the average power of a random signal in wireless sensor networks. Due to stringent bandwidth/power constraints, each sensor quantizes its observation into one bit of information and sends the quantized data to a fusion center, where the signal power is estimated. We firstly introduce two fixed quantization (FQ) schemes, with the first using a single threshold and the second employing a pair of symmetric thresholds. The maximum likelihood (ML) estimators associated with the two FQ schemes are developed and their corresponding Cramer-Rao bounds (CRBs) are analyzed. We show that the FQ approach, especially the second one, can achieve an estimation performance close to that of a clairvoyant estimator using unquantized data, if the optimum quantization threshold is available; however, the optimum threshold is dependent on the unknown signal power and as the threshold deviates from its optimum value, the performance degrades rapidly. To cope with this difficulty, we propose a distributed adaptive quantization (AQ) approach by which the threshold is dynamically adjusted from one sensor to another, in a way such that the threshold converges to the optimum threshold. Our analysis shows that the proposed AQ approach is asymptotically optimum, yielding an asymptotic CRB equivalent to that of the FQ approach with optimum threshold.

**Index Terms**—Distributed estimation, adaptive quantization, wireless sensor network (WSN).

## I. INTRODUCTION

Recent developments in computing and wireless communication technology have led to the emergence of small, inexpensive sensors capable of sensing, processing, and communication. A wireless sensor network (WSN) consisting of a large number of such sensors is able to accomplish a variety of tasks including environment monitoring, battlefield surveillance, target localization and tracking, and many more [1], [2]. Distributed parameter estimation is one of the fundamental problems arising from the wide applications of WSNs.

Since sensors in a network are powered by small-size batteries whose energy resource is severely limited, energy constraints are a primary issue that needs to be taken into account in designing distributed estimation algorithms. A multitude of studies along this line have appeared recently, e.g. distributed estimation in the context of decentralized compression-estimation [3], [4], optimal resource allocation [5]–[7], and distributed estimation in the framework of sensor cooperation [8], [9]. Meanwhile, some other works [10]–[17] considered distributed estimation using aggressive quantization

strategies, aimed to address not only the energy constraint, but also the bandwidth constraint which is inherent in WSNs. In this setup, quantization becomes an integral part of the estimation process and is critical to the estimation performance. Different quantization schemes were proposed to attain an acceptable estimation accuracy while meeting the stringent power/bandwidth budgets.

Specifically, by modeling the unknown parameter as a random parameter, Bayesian techniques were proposed in, e.g., [10], [18], [19]. These methods require knowledge of the joint distribution of the unknown parameter and the observed signals for quantizer design. Another category of methods treat the unknown parameter as a deterministic unknown parameter. A notable example is a fixed quantization (FQ) approach, where a common quantization threshold  $\tau$  is applied at all sensors [11], [12]. The drawback of the FQ approach is that its estimation performance is sensitive to the quantization threshold, whose choice is always a tricky problem in practice. A remedy is to employ multiple thresholds instead of one threshold, hoping that one of the thresholds is close to the optimum value. In [11], the authors proposed to periodically apply one of a set of thresholds and each threshold is employed with equal frequencies (through a periodic control signal or dithering added before quantization). Also, in [12], an unequal-frequency multi-thresholding strategy was developed, which allows some thresholds (in particular those statistically closer to the optimum threshold) being used more frequently than the others. Another recent method addressing quantization with deterministically unknown parameter was introduced in [16], where the idea is to optimize the worse-case performance by maximizing the minimum asymptotic efficiency between two maximum likelihood (ML) estimators using quantized and, respectively, unquantized observations.

Most studies on distributed quantization and estimation, including [10]–[17], however, considered the estimation of a *mean* or *location* parameter under an additive model:  $x_n = \theta + w_n$ , where  $x_n$  denotes the unquantized sensor observation made at sensor  $n$ ,  $\theta$  is the unknown mean parameter, and  $w_n$  is the sensor noise with zero mean and known distribution. While the above model covers a range of important applications, we are interested in a different problem arising from other applications such as spectrum sensing whose objective is energy detection and estimation. The problem is to estimate a *scale* parameter associated with the sensor observations. Specifically, suppose we have  $N$  spatially distributed sensors, each sensor making an independent and identically distributed (i.i.d.) observation  $x_n$  from a certain distribution,  $p_X(x)$ , with zero mean and unknown variance  $\sigma^2$ .

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The problem of interest is to design one-bit quantization strategies,  $\{Q_n(\cdot)\}$ , to convert  $\{x_n\}$  into binary data  $\{b_n\}$  which are forwarded to a fusion center (FC), and to find an effective estimate of the standard deviation or scale parameter,  $\sigma$ , from  $\{b_n\}$  at the FC. Such a problem finds important applications, for example, in cognitive radios where a group of secondary users collaboratively measure the power of a primary user signal for opportunistic spectrum usage [20]–[25], and in many other sensor network applications such as detection and estimation which need to collect the statistics of a signal/observation noise for the algorithm design, e.g. [26], [27]. When a quantization strategy is given, maximum likelihood (ML) estimation of  $\sigma$  using quantized data was considered in [28]. In this paper, we consider joint quantization and estimation, examine the impact of quantization on the estimation performance, and develop a new adaptive quantization (AQ) approach for the estimation of  $\sigma$ .

Specifically, two fixed quantization (FQ) schemes are firstly introduced in this paper, where a single threshold and a pair of symmetric thresholds are employed, respectively. Theoretical analysis shows that the FQ scheme using dual thresholds has a better estimation performance, yielding a Cramér-Rao bound (CRB) that is about one half of that of the FQ scheme with a single threshold. Also, by choosing an optimum quantization threshold, both FQ schemes are able to achieve an estimation performance close to that of a ML estimator using unquantized data (also referred to as “clairvoyant estimator” in this paper). Specifically, for Gaussian distribution, the estimation variance of the FQ with a single threshold is within about 3 times that of the clairvoyant estimator, and the estimation variance of the FQ with a single threshold is within about 1.5 times that of the clairvoyant estimator.

Although the FQ approach provides a comparable performance to the clairvoyant estimator while requiring only one bit information from each sensor, its problem lies in that the optimum quantization threshold is dependent on the unknown parameter to be estimated, which is not usable in practice. Also, as the threshold deviates from its optimum value, its performance drops rapidly. To cope with this difficulty, we propose an adaptive quantization (AQ) approach which, with sensors *sequentially* broadcasting their quantized data, allows each sensor to adaptively adjust its quantization threshold. We design our AQ scheme by resorting to the ML estimator and a relationship between the optimum threshold and the unknown parameter found by an analysis. Our analysis shows that our proposed AQ scheme is asymptotically optimum, which yields an asymptotic CRB equivalent to that of the FQ approach with optimum threshold.

Note that our AQ scheme here can be considered as an extension of [17] to a scale parameter estimation problem. This extension, however, is not that straightforward and yields many interesting results. Firstly, the fundamental approach of [17] and the resulting optimality is tied to the notion that as the quantization threshold approaches the parameter to be estimated, the performance of the estimation approaches the best possible. This fact may not be true for general parameter estimation problems. For example, for the scale parameter estimation in this paper, the relationship between optimal

threshold and the parameter to be estimated is non-trivial and needs to be found out by numerical search. In this case, our results show that the AQ approach can be easily extended to incorporate this generalized relationship and the asymptotic optimality still remains true. Secondly, the technical analysis, especially the asymptotic performance analysis of AQ-ML scheme, of [17] is restricted to the Gaussian noise setup. In this paper, we have relaxed this restriction and it is shown that the asymptotic optimality of the proposed AQ approach holds for any continuous noise distribution.

The rest of the paper is organized as follows. Two fixed quantization schemes are introduced in Section II with their MLEs developed and CRB analyses carried out. In Section III, an adaptive quantization approach is proposed, and its asymptotic performance analysis is derived. Numerical results and comparisons are presented in Section IV, followed by concluding remarks in Section V.

## II. FIXED QUANTIZATION

### A. Fixed Quantization: Single Threshold

As in [11], [12], we employ a common threshold  $\tau$  for all sensors to quantize the observations into one-bit information:

$$b_n = \text{sgn}(x_n - \tau), \quad n = 1, 2, \dots, N, \quad (1)$$

where

$$\text{sgn}\{x\} = \begin{cases} 0, & \text{if } x \leq 0, \\ 1, & \text{if } x > 0. \end{cases}$$

To facilitate our analysis, we express  $x_n$  as

$$x_n = \sigma v_n, \quad n = 1, 2, \dots, N \quad (2)$$

where  $v_n$  denotes a random variable having the same distribution as  $x_n$  but with zero mean and *unit* variance,  $\sigma$  is the unknown *scale* parameter to be estimated. It can be readily shown that the probability mass function (PMF) of  $b_n$  is given by

$$P(b_n; \sigma) = (1 - F_V(\tau/\sigma))^{b_n} (F_V(\tau/\sigma))^{1-b_n} \quad (3)$$

where  $p_V(x)$  and  $F_V(x)$  denote the probability density function (PDF) and the cumulative distribution function (CDF) of  $v_n$ , respectively. Since  $\{b_n\}$  are i.i.d., the log-PMF or log-likelihood function is

$$\begin{aligned} L_{\text{FQS}}(\sigma) &\triangleq \log[P(b_1, \dots, b_N; \sigma)] \\ &= \sum_{n=1}^N \left\{ b_n \log[1 - F_v(\tau/\sigma)] + (1 - b_n) \log[F_v(\tau/\sigma)] \right\} \end{aligned} \quad (4)$$

where we use the subscript FQS (FQ with a Single threshold) to represent the current FQ scheme. The ML estimate and CRB associated with this scheme are given in the following proposition.

*Proposition 1:* For the FQS scheme, the ML estimate of  $\sigma$  is given by

$$\hat{\sigma} = \frac{\tau}{F_V^{-1}\left(1 - (\sum_{n=1}^N b_n)/N\right)} \quad (5)$$

where  $F_V^{-1}$  denotes the inverse of the CDF. Furthermore, the CRB for any unbiased estimator based on  $\{b_n\}$  is

$$\text{CRB}_{\text{FQS}}(\sigma) = \frac{1}{N} \frac{\sigma^4 F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))}{\tau^2 p_V^2(\tau/\sigma)}. \quad (6)$$

*Proof:* See Appendix A. ■

We see that  $\text{CRB}_{\text{FQS}}(\sigma)$  depends on the quantization threshold  $\tau$ . To find an optimum  $\tau$ , we rewrite (6) as

$$\begin{aligned} \text{CRB}_{\text{FQS}}(\sigma) &= \frac{\sigma^2}{N} \frac{\sigma^2 F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))}{\tau^2 p_V^2(\tau/\sigma)} \\ &= \frac{\sigma^2}{N} \frac{1}{\gamma^2} \frac{F_V(\gamma)(1 - F_V(\gamma))}{p_V^2(\gamma)} \\ &\triangleq \frac{\sigma^2}{N} G_{\text{FQS}}(\gamma) \end{aligned} \quad (7)$$

where  $\gamma \triangleq \tau/\sigma$  denotes the ratio of the threshold to the unknown parameter to be estimated. To minimize the CRB, the optimum  $\gamma$  is the one minimizing the function  $G_{\text{FQS}}(\gamma)$ .

Specifically, for the Gaussian distribution, the optimum  $\gamma$  is about  $\pm 1.57$  (see Fig. 1). Hence the optimum quantization threshold is  $\pm 1.57\sigma$  for the Gaussian distribution. To better evaluate the performance of the FQS scheme, we compare it with the ML estimator using *unquantized* data (also referred to as “clairvoyant estimator”), which provides a lower bound on the achievable estimation performance of all rate-constrained methods, and serves as a benchmark for evaluating the efficiency of the proposed quantization schemes. It is easy to derive (the derivation is straightforward and hence omitted here) that for the Gaussian observations  $\{x_n\}$ , the CRB for any unbiased estimator based on the unquantized data  $\{x_n\}$  is given as

$$\text{CRB}_{\text{NQ}}(\sigma) = \frac{\sigma^2}{2N}. \quad (8)$$

where we use the subscript NQ to stand for no quantization. Clearly, we see that the minimal CRB achieved by the FQS scheme using the optimum quantization threshold is only about  $2G_{\text{FQS}}(1.57) \approx 3$  times that of the clairvoyant estimator using unquantized data. Nevertheless, from Fig. 1, we observe that the performance of the FQS scheme degrades rapidly as the threshold  $\tau$  deviates from its optimum value  $1.57\sigma$ . Note that without any prior information of the true  $\sigma$ , the optimum choice of the quantization threshold is arbitrary because the optimum threshold minimizing the CRB is dependent on the unknown parameter  $\sigma$ .

### B. Fixed Quantization: A Pair of Symmetric Thresholds

Our previous analysis for FQS (i.e., the CRB is an even function of the threshold) motivates us to consider a symmetric quantization scheme using a pair of symmetric thresholds  $\pm\tau$ , which is defined as

$$b_n = \text{sgn}(|x_n| - \tau) = \begin{cases} 0 & \text{if } -\tau \leq x_n \leq \tau \\ 1 & \text{otherwise} \end{cases}. \quad (9)$$

Intuitively, this quantization scheme is able to achieve a better performance as compared with the FQS scheme because the quantized bit,  $b_n$ , reveals more information about the signal variance by locating the absolute value of the observation. For

this dual thresholds based quantizer, the PMF of  $b_n$  is given as

$$P(b_n; \sigma) = (2 - 2F_V(\tau/\sigma))^{b_n} (2F_V(\tau/\sigma) - 1)^{1-b_n}. \quad (10)$$

It follows that the log-likelihood function is

$$\begin{aligned} L_{\text{FQD}}(\sigma) &= \sum_{n=1}^N \left[ b_n \log [2 - 2F_V(\tau/\sigma)] \right. \\ &\quad \left. + (1 - b_n) \log [2F_V(\tau/\sigma) - 1] \right] \end{aligned} \quad (11)$$

where the subscript FQD (Fixed Quantization with Dual thresholds) represents the current scheme. We have the following result regarding its ML estimate and CRB.

*Proposition 2:* For the FQD scheme, the ML estimate of  $\sigma$  is given by

$$\hat{\sigma} = \frac{\tau}{F_V^{-1} \left( 1 - (\sum_{n=1}^N b_n) / (2N) \right)}. \quad (12)$$

The CRB for any unbiased estimator based on  $\{b_n\}$  is given by

$$\text{CRB}_{\text{FQD}}(\sigma) = \frac{1}{2N} \frac{\sigma^4 (1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)}{\tau^2 p_V^2(\tau/\sigma)}. \quad (13)$$

*Proof:* See Appendix B. ■

As we did for the single threshold case, we can rewrite (13) as

$$\begin{aligned} \text{CRB}_{\text{FQD}}(\sigma) &= \frac{\sigma^2}{2N} \frac{1}{\gamma^2} \frac{(1 - F_V(\gamma))(2F_V(\gamma) - 1)}{p_V^2(\gamma)} \\ &\triangleq \frac{\sigma^2}{2N} G_{\text{FQD}}(\gamma) \end{aligned} \quad (14)$$

where  $\gamma \triangleq \tau/\sigma$ .

Specifically, for the Gaussian distribution, the optimum  $\gamma$  minimizing the CRB is about 1.48 (see Fig. 1). Consequently the optimum threshold  $\tau$  is  $1.48\sigma$  for the Gaussian distribution and the corresponding minimal CRB achieved is only about  $G_{\text{FQD}}(1.48) \approx 1.5$  times that of the clairvoyant estimator (cf. (8)). Also, from Fig. 1, we can see that the FQD scheme outperforms the FQS scheme at all thresholds. This can be intuitively justified since the FQD scheme produces a binary bit that contains more information about the observation and the unknown parameter associated with the observations.

## III. ADAPTIVE QUANTIZATION

As we can see from previous analyses, both FQ schemes are very sensitive to the choice of the quantization threshold  $\tau$ : the estimation performance of the FQ schemes degrades sharply as  $\tau$  deviates from their optimum values. However, the optimum threshold is dependent on the unknown parameter  $\sigma$  to be estimated, which is not usable in practice. To cope with this difficulty, we propose a data-dependent distributed adaptive quantization (AQ) approach by which the threshold is dynamically adjusted from one sensor to another, in a way such that the threshold converges to the optimum threshold. We adopt the following assumptions for the AQ approach:

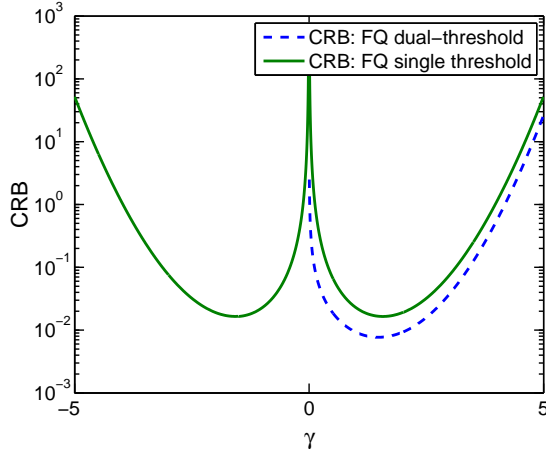


Fig. 1: Gaussian observations: CRBs of the FQS and FQD schemes vs.  $\gamma$ ,  $N = 100$ .

- A1 We assume each sensor sends its quantized data to the FC sequentially with the help of a scheduling algorithm, e.g. [29].
- A2 While each sensor transmits, the other sensors can listen to the transmission due to the broadcasting nature of the wireless channel. To focus on the quantization problem, we assume that the quantized data are received without errors (by using, e.g., a strong error correction code).

A detailed discussion of these two assumptions will be provided in Section III-C.

#### A. AQ

For the AQ approach, each sensor, say sensor  $n$ , finds its quantization threshold,  $\tau_n$ , by using the quantized data  $\{b_k\}_{k=1}^{n-1}$  received from previous sensors. We firstly employ the ML estimator to compute  $\hat{\sigma}_n$ , where  $\hat{\sigma}_n$  denotes an estimate of  $\sigma$  at sensor  $n$  based on  $\{b_k\}_{k=1}^{n-1}$ . The threshold  $\tau_n$  is then calculated according to the  $\tau_{\text{opt}} \sim \sigma$  relationship established by the FQ analyses, e.g., for Gaussian observations,  $\tau_{\text{opt}} = 1.57\sigma$  if a single threshold quantization scheme is adopted or  $\tau_{\text{opt}} = 1.48\sigma$  if a pair of symmetric thresholds are adopted. In this section, we only consider the AQ approach employing a pair of symmetric thresholds, i.e. each sensor quantizes its observation using the form of (9), as it yields better estimation performance. The details of the AQ scheme is described as follows.

We firstly generate two quantized bits  $b_1$  and  $b_2$  for initialization. For sensor 1, we use an arbitrary positive threshold, say  $\tau_1 = 1$ , to generate  $b_1$ :

$$b_1 = \text{sgn}(|x_1| - \tau_1). \quad (15)$$

Then,  $b_1$  is sent to the FC and all other sensors. Upon receiving  $b_1$ , sensor 2 computes  $\tau_2 = \tau_1 \Delta^{b_1} \Delta^{b_1-1}$ , that is,  $\tau_2 = \tau_1 \Delta$  if  $b_1 = 1$  and  $\tau_2 = \tau_1 / \Delta$  if  $b_1 = 0$ , and uses it to generate  $b_2$ , where  $\Delta$  is a stepsize whose choice will be discussed shortly. Also, we assume that the initial threshold  $\tau_1$  and the stepsize  $\Delta$  are known to all sensors. Based on the received  $\{b_1, b_2\}$ ,

sensor 3 finds the ML estimate of  $\sigma$  as

$$\hat{\sigma}_3 = \arg \max_{\sigma} L_3(\sigma; b_1, b_2, \tau_1, \tau_2) \quad (16)$$

where

$$L_3(\sigma; b_1, b_2, \tau_1, \tau_2) = \sum_{k=1}^2 \left[ b_k \log [2 - 2F_V(\tau_k/\sigma)] + (1 - b_k) \log [2F_V(\tau_k/\sigma) - 1] \right] \quad (17)$$

denotes the log-likelihood function of  $\sigma$  given binary observations  $b_1, b_2$  and the associated thresholds  $\tau_1, \tau_2$ , where  $\tau_2$  can be recovered from  $\tau_2 = \tau_1 \Delta^{b_1} \Delta^{b_1-1}$ . The stepsize  $\Delta$  used by sensor 2 should be large enough such that  $b_1$  and  $b_2$  have different discrete values. Otherwise, it can be shown that  $\hat{\sigma}_3$  obtained above is either infinity or zero (depending on the values of  $b_1$  and  $b_2$ ), which should be avoided. Although there is always a non-zero probability for  $b_1$  and  $b_2$  to have identical values, the probability can be made practically small by choosing  $\Delta$  sufficiently large. In addition, if for a chosen  $\Delta$ , the first two quantized bits are still of an identical value, the following sensors can keep adjusting the threshold by  $\tau_{n+1} = \tau_n \Delta^{b_n} \Delta^{b_n-1}$  until a binary bit of a different value is generated, at which point the quantization process is switched to use the ML estimator.

The threshold  $\tau_3$  is then computed as

$$\tau_3 = \mu \hat{\sigma}_3 \quad (18)$$

where  $\mu$  is the coefficient of the relationship between the optimum threshold  $\tau_{\text{opt}}$  and the unknown parameter  $\sigma$  for the corresponding FQ approach. Here  $\mu$  is chosen to minimize (14) since a pair of symmetric thresholds are used here.

In general, for sensor  $n$ , it first recovers the previous thresholds  $\{\tau_k\}_{k=1}^{n-1}$  from the received quantized data  $\{b_k\}_{k=1}^{n-1}$ , which can be computed straightforwardly by the following recursive calculation:

$$\begin{aligned} \tau_2 &= \tau_1 \Delta^{b_1} \Delta^{b_1-1} \\ \tau_3 &= \mu \hat{\sigma}_3 & \hat{\sigma}_3 &= \arg \max_{\sigma} L_3(\sigma; b_1, b_2, \tau_1, \tau_2) \\ &\vdots \\ \tau_{n-1} &= \mu \hat{\sigma}_{n-1} & \hat{\sigma}_{n-1} &= \arg \max_{\sigma} L_{n-1}(\sigma; \{b_k\}_{k=1}^{n-2}, \{\tau_k\}_{k=1}^{n-2}). \end{aligned} \quad (19)$$

After obtaining  $\{\tau_1, \tau_2, \dots, \tau_{n-1}\}$ , sensor  $n$  computes its current threshold  $\tau_n$  as  $\tau_n = \mu \hat{\sigma}_n$ , with  $\hat{\sigma}_n$  given by

$$\hat{\sigma}_n = \arg \max_{\sigma} L_n(\sigma; \{b_k\}_{k=1}^{n-1}, \{\tau_k\}_{k=1}^{n-1}) \quad (20)$$

where

$$L_n(\sigma; \{b_k\}_{k=1}^{n-1}, \{\tau_k\}_{k=1}^{n-1}) = \sum_{k=1}^{n-1} \left[ b_k \log [2 - 2F_V(\tau_k/\sigma)] + (1 - b_k) \log [2F_V(\tau_k/\sigma) - 1] \right] \quad (21)$$

is the log-likelihood function of  $\sigma$  given  $\{b_k\}_{k=1}^{n-1}$ .

The ML estimator at the FC to find the final estimate of  $\sigma$  from the received quantized data  $\{b_1, b_2, \dots, b_N\}$  is given by

$$\hat{\sigma} = \arg \max_{\sigma} L_{\text{AQ}}(\sigma; \{b_k\}_{k=1}^N, \{\tau_k\}_{k=1}^N) \quad (22)$$

where all thresholds  $\{\tau_1, \dots, \tau_N\}$  can be recovered from the quantized data  $\{b_1, b_2, \dots, b_{N-1}\}$  in a recursive way described above. Note that unlike the FQ schemes, the ML estimators (21) and (22) generally admit no closed-form solution, and a searching algorithm has to be utilized. Nevertheless, the computational complexity is moderate since only one-dimensional search is involved.

### B. CRB

We evaluate the performance of the proposed AQ approach through analysis of the corresponding Cramér-Rao bound (CRB), a lower bound on the mean-squared error (MSE) that is asymptotically achieved by the MLE (22). By following the same derivation as in Appendix B, it can be easily verified that the second-order derivative of  $L_{\text{AQ}}(\sigma)$  is

$$\begin{aligned} \ddot{L}_{\text{AQ}}(\sigma) &= \frac{1}{\sigma^2} \sum_{n=1}^N \left\{ \frac{-2\frac{\tau_n}{\sigma} p_V(\tau_n/\sigma)(b_n - 2 + 2F_V(\tau_n/\sigma))}{(1 - F_V(\tau_n/\sigma))(2F_V(\tau_n/\sigma) - 1)} \right. \\ &\quad + \frac{-2\frac{\tau_n^2}{\sigma^2} p_V^2(\tau_n/\sigma) + \tau_n(b_n - 2 + 2F_V(\tau_n/\sigma)) \dot{p}_V(\tau_n/\sigma)}{(1 - F_V(\tau_n/\sigma))(2F_V(\tau_n/\sigma) - 1)} \\ &\quad \left. - \frac{\frac{\tau_n^2}{\sigma^2} p_V^2(\tau_n/\sigma)(4F_V(\tau_n/\sigma) - 3)(b_n - 2 + 2F_V(\tau_n/\sigma))}{[(1 - F_V(\tau_n/\sigma))(2F_V(\tau_n/\sigma) - 1)]^2} \right\} \\ &\triangleq \sum_{n=1}^N A(b_n, \tau_n, \sigma). \end{aligned} \quad (23)$$

The Fisher information is given by

$$\begin{aligned} J_{\text{AQ}}(\theta) &= -E \left[ \ddot{L}_{\text{AQ}}(\sigma) \right] \\ &= -\sum_{n=1}^N E_{b_n, \tau_n} [A(b_n, \tau_n, \sigma)] \end{aligned} \quad (24)$$

where  $E_{b_n, \tau_n}$  denotes the expectation with respect to the joint distribution of  $b_n$  and  $\tau_n$  (note that  $\{\tau_n\}$  are also random variable as they are determined by  $\{b_n\}$ ). Since

$$P(b_n, \tau_n; \sigma) = P(\tau_n; \sigma)P(b_n|\tau_n; \sigma) \quad (25)$$

we can write

$$\begin{aligned} J_{\text{AQ}}(\sigma) &= -\sum_{n=1}^N E_{\tau_n} \left\{ E_{b_n|\tau_n} [A(b_n, \tau_n, \sigma)] \right\} \\ &\stackrel{(a)}{=} \sum_{n=1}^N E_{\tau_n} \left[ 2\frac{\tau_n^2}{\sigma^4} \frac{p_V^2(\tau_n/\sigma)}{(1 - F_V(\tau_n/\sigma))(2F_V(\tau_n/\sigma) - 1)} \right] \\ &\stackrel{(b)}{=} \frac{2}{\sigma^2} \sum_{n=1}^N \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \end{aligned} \quad (26)$$

where  $E_{\tau_n}$  denotes the expectation with respect to the distribution  $P(\tau_n; \sigma)$ <sup>1</sup>,  $E_{b_n|\tau_n}$  denotes the expectation with respect to the conditional distribution  $P(b_n|\tau_n; \sigma)$ , (a) follows

<sup>1</sup>As we will discuss shortly after, the threshold  $\tau_n$  is a discrete random variable. However, we still treat it as a continuous random variable by using the dirac-delta function so that we can present our expressions more conveniently by using the integral operator.

from the fact that  $b_n$  is a binary random variable with  $P(b_n = 1|\tau_n, \sigma) = 2 - 2F_V(\tau_n/\sigma)$  and  $P(b_n = 0|\tau_n, \sigma) = 2F_V(\tau_n/\sigma) - 1$ , and we define

$$G(\tau_n; \sigma) \triangleq \frac{\sigma^2 (1 - F_V(\tau_n/\sigma))(2F_V(\tau_n/\sigma) - 1)}{\tau_n^2 p_V^2(\tau_n/\sigma)} \quad (27)$$

in (b).

To compute the exact Fisher information (26), we need to determine the distributions of  $\{\tau_n\}$ , i.e.  $\{P(\tau_n; \sigma)\}$ . Since the ML estimator used to find the threshold is a nonlinear function, the threshold  $\tau_n$  is a discrete random variable with the number of possible values for  $\tau_n$  increases exponentially with  $n$ . Specifically, sensor  $n$  has  $2^{n-1}$  possible threshold values with each value chosen with a certain probability. The exact computation of  $P(\tau_n; \sigma)$  is, therefore, cumbersome, especially when the number of sensors,  $N$ , is large. To circumvent the difficulty in computing the exact  $P(\tau_n; \sigma)$ , we examine the asymptotic performance which offers additional insight into the AQ scheme.

*Proposition 3:* For continuous noise distribution  $p_V(x)$ , as  $N$  increases, the CRB of the proposed AQ scheme converges to the CRB of the FQ scheme using the optimum threshold, i.e.

$$N\text{CRB}_{\text{AQ}}(\sigma) \rightarrow N\text{CRB}_{\text{FQD}}(\tau_{\text{opt}}; \sigma). \quad (28)$$

*Proof:* See Appendix C. ■

Note that we multiply the CRBs on both sides of (28) by a factor  $N$  because we have to properly normalize the CRBs, otherwise both terms vanish with an increasing  $N$ , and the claim loses its meaning. This result indicates that our AQ scheme adaptively finds the best threshold by learning from prior transmissions. Without any prior knowledge of the unknown parameter, the proposed AQ scheme is able to asymptotically achieve a CRB which is attained by the FQD scheme with an optimum threshold.

### C. Discussions

As compared with the FQ schemes, the AQ scheme involves a scheduling policy and needs the sensors to spend more power in reception and computation. The design of transmission scheduling in wireless sensor networks has been addressed in many works, e.g. [29] and the references therein. The FC can be used to collect the statistics of the data and to develop a scheduling algorithm. Also, the scheduling algorithm can be dominated by the FC to relieve the sensors from transmitting any overhead information for scheduling. For the power consumption issue, it is suggested in [30] that in a sensor network, communication consumes a significant portion (about 70%) of the total energy, whereas reception, sensing and computation consume only a small portion of the total energy. Hence, considering the performance gain the AQ approach achieves relative to the FQ schemes, the additional reception and computation power consumed by the AQ scheme may be acceptable in many scenarios.

In order to focus on the quantization problem, we assumed that the quantized data are received without errors for our approach (see assumption (A2)). Imperfect communication due to noisy channels and limited transmission power will

affect the performance of all distributed estimation schemes, including ours. Albeit important, we consider this a separate issue. Moreover, in practice, we can minimize the adverse effect of imperfect communication by implementing the AQ approach in a proper manner. For example, an effective way is to let the FC dominate the whole process. The sensors no longer need to hear from other sensors and compute the quantization threshold, instead, the FC keeps track of the quantization threshold as in (19–21) (assuming energy budget for the FC is not a major issue). Each sensor wakes up only when polled by the FC; during this process (polling), the FC assigns the quantization threshold to each sensor sequentially and the polled sensor, based on the assigned quantization threshold, generates its quantized data and reports back to the FC. This FC-dominated AQ can effectively suppress the imperfect communication problem since it involves communication only between the FC and the sensors. Of course, the channel links from the sensors to the FC may still be unreliable due to the sensors’ limited transmission power (the links from the FC to the sensors can be considered ideal as the power of the FC is not a major issue). However, as long as the transmission error probability (from the sensors to the FC) is kept low, the AQ approach is expected to be robust to the communication errors and the resultant error propagation because each quantization threshold is computed by the ML estimator at the FC based on all previous quantization thresholds (these thresholds are exactly known at the FC as they are calculated by the FC) and all quantized data received from previous sensors, and therefore the computed quantization threshold will still keep the tendency towards the optimal threshold.

#### IV. SIMULATION RESULTS

In this section, we illustrate the performance of the FQ and AQ schemes. Two examples are considered, where the observations  $\{x_n\}$  are assumed i.i.d. Gaussian random variables and i.i.d. Laplace random variables (also known as double exponential distribution), respectively.

##### A. Gaussian Observations

We firstly examine the information loss of the FQ and AQ schemes relative to the ML estimator using unquantized data. The concept “information loss” is borrowed from [11], which is defined as the ratio (in dB) of the CRB for the proposed scheme to the CRB for the clairvoyant estimator using unquantized data:

$$\text{IL} \triangleq 10 \log_{10} \frac{\text{CRB}_{\text{Q-based}}(\sigma)}{\text{CRB}_{\text{NQ}}(\sigma)} \quad (29)$$

where we use the subscript Q-based to represent any quantization scheme. Note that although, for the AQ scheme, an exact computation of the CRB is impossible, nevertheless, (26) can still be evaluated numerically by Monte Carlo integration. We set  $\sigma = 1$ . Fig. 2 shows the information loss of the FQ and AQ schemes as a function of the number of sensors,  $N$ . It can be seen that the information loss of the FQ schemes is independent of the number of sensors,  $N$ . Also, when the optimum thresholds are used, i.e.  $\tau = 1.57$  for FQS

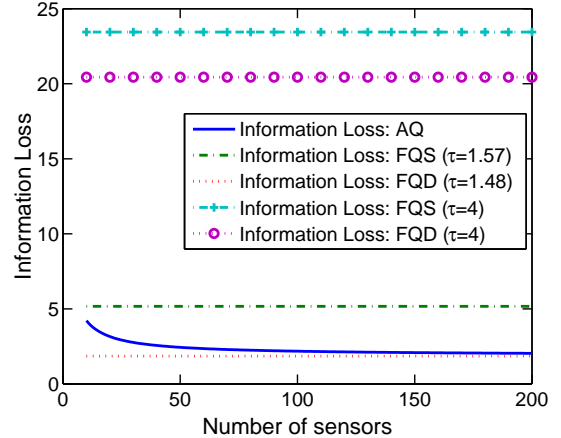


Fig. 2: Gaussian observations: Information loss of the FQ and AQ schemes relative to the ML estimator using unquantized data,  $\sigma = 1$ .

and  $\tau = 1.48$  for FQD, the FQ schemes incur a moderate information loss, which is about 5dB for FQS and 2dB for FQD. However, the FQ schemes are very sensitive to the value of  $\tau$ ; as the threshold  $\tau$  becomes more apart from the optimum value (even not too far apart, e.g.  $\tau = 4$ ), the performance of the FQ schemes degrades significantly. As for the AQ scheme, the information loss decreases with an increasing  $N$ . This is because the AQ scheme benefits from the previous transmissions by adaptively choosing a proper quantization threshold. Also, we observe that the information loss of the AQ scheme approaches that of the FQD scheme with optimum threshold, i.e.  $\tau = 1.48$ , which corroborates our previous claim in Proposition 3.

The mean square errors (MSEs) of the ML estimators for the FQD and AQ schemes are included and compared with the corresponding CRB in Fig. 3, where we set  $\sigma = 1$ . For the AQ scheme and the FQD scheme with optimum threshold  $\tau = 1.48$ , it is observed that the MSEs approach the CRBs within a moderate number of sensors,  $N$ . However, this is not true for the FQD scheme with a non-optimum threshold  $\tau = 3$ . In this case, the ML estimator needs much more sensors to converge to its corresponding CRB. As we also see from this figure, the performance of the AQ scheme approaches that of the FQD with optimum threshold ( $\tau = 1.48$ ) while without knowing any prior information of the unknown parameter  $\sigma$ .

We plot the MSEs of the ML estimators for the FQ schemes as a function of  $\gamma = \tau/\sigma$  in Fig. 4, where we set  $N = 100$  and  $\sigma = 1$ . It is seen that the ML estimators achieves its asymptotic performance with moderate number of sensors ( $N = 100$ ) when the ratio  $\gamma$  is around its optimum value.

Our analysis shows that the performance of the FQ schemes is dependent on the value of the unknown parameter  $\sigma$ , even if the ratio  $\gamma = \tau/\sigma$  is fixed (cf. (6) and (14)). Specifically, the CRBs are proportional to  $\sigma^2$ , indicating that a smaller  $\sigma$  results in a better performance. Such a relationship also applies for the clairvoyant estimator and the AQ scheme, which can be easily observed from their CRB expressions. The performance of the respective schemes versus the unknown parameter is plotted

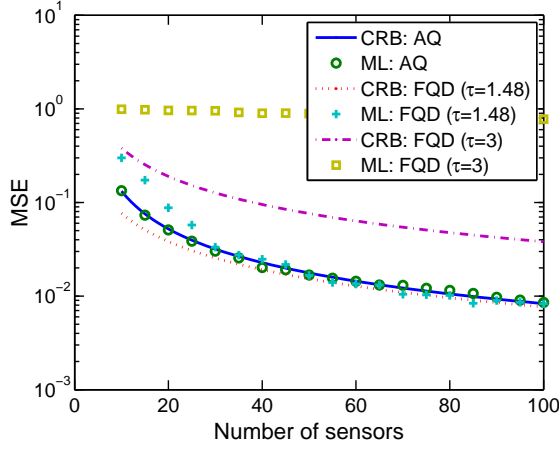


Fig. 3: Gaussian observations: MSEs and CRBs of the FQD and AQ schemes,  $\sigma = 1$ .

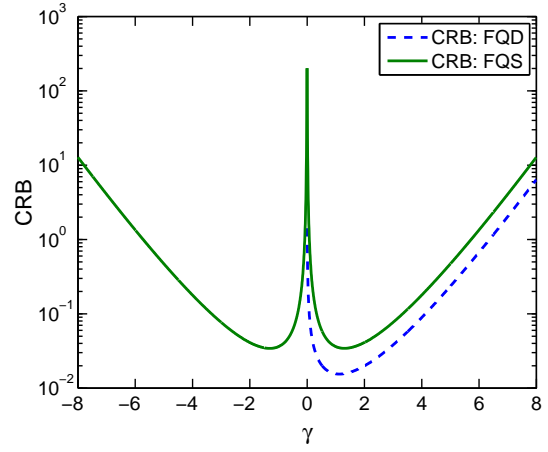


Fig. 6: Laplace observations: CRBs of the FQS and FQD schemes vs.  $\gamma$ ,  $N = 100$ ,  $\sigma = 1$ .

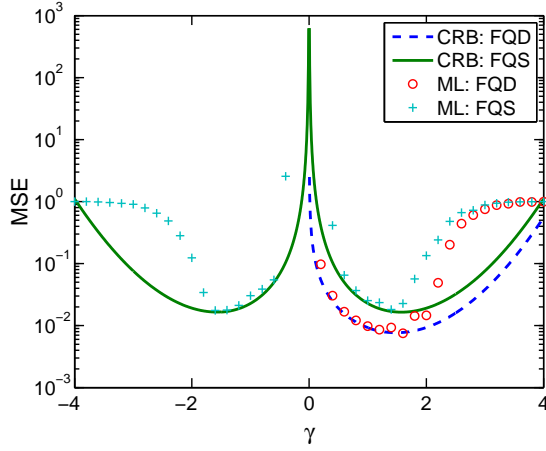


Fig. 4: Gaussian observations: MSEs and CRBs of the FQ schemes versus  $\gamma$ ,  $\sigma = 1$ ,  $N = 100$ .

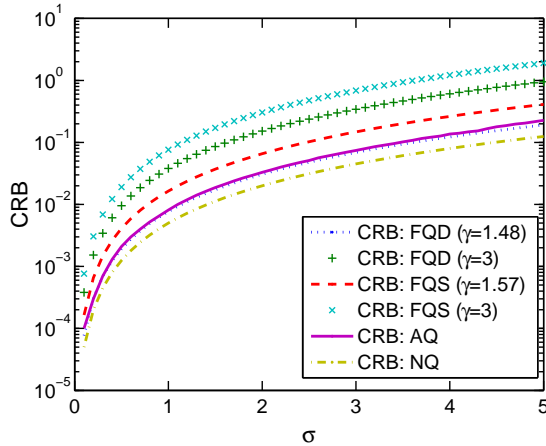


Fig. 5: Gaussian observations: CRBs of the respective schemes versus the unknown parameter  $\sigma$ ,  $N = 100$ .

in Fig. 5, where  $\gamma$  is chosen to be 1.48 and 3 for the FQD scheme, and 1.57 and 3 for the FQS scheme, respectively, the number of sensors,  $N$ , is set to be 100.

### B. Laplace Observations

We consider the case where the observations  $\{x_n\}$  follow a Laplace distribution with zero mean and variance  $\sigma^2$ , i.e.

$$p_X(x_n) = \frac{\sqrt{2}}{2\sigma} \exp\left(-\frac{\sqrt{2}|x_n|}{\sigma}\right) \quad (30)$$

Fig. 6 depicts the CRBs of the FQ schemes vs.  $\gamma$ , the ratio of the quantization threshold  $\tau$  to the unknown parameter  $\sigma$  to be estimated. As compared with Fig. 1, we see that the FQ schemes demonstrate a similar behavior for both Gaussian and Laplace distributions: in both cases the FQ approach suffers from a rapid performance loss when the threshold deviates from the optimum value. A numerical search finds that for the Laplace observations, the optimum threshold is  $1.30\sigma$  for the FQS scheme and  $1.125\sigma$  the FQD scheme.

We now examine the information loss of the FQ and AQ schemes relative to the ML estimator using unquantized data. It is easy to derive (the derivation is straightforward and hence omitted here) that for the Laplace observations, the CRB for any unbiased estimator based on the unquantized data  $\{x_n\}$  is given by

$$\text{CRB}_{\text{NQ}}(\sigma) = \frac{\sigma^2}{N}. \quad (31)$$

Fig. 7 shows the information loss of the FQ and AQ schemes as a function of the number of sensors,  $N$ , where we set  $\sigma = 1$ . Again, the AQ approach achieves an asymptotic optimum performance with an increasing  $N$ , irrespective of the observation distributions.

### V. CONCLUSION

The problem of power estimation from multi-sensors' observations was considered in the paper. In particular, we assume each sensor makes an independent observation from a certain distribution with zero mean and unknown variance. The objective is to estimate the standard deviation associated with the distribution in bandwidth/power constrained wireless sensor networks (WSNs). Two fixed quantization (FQ) schemes and an adaptive quantization (AQ) scheme were proposed and their corresponding MLEs were developed. CRB analyses

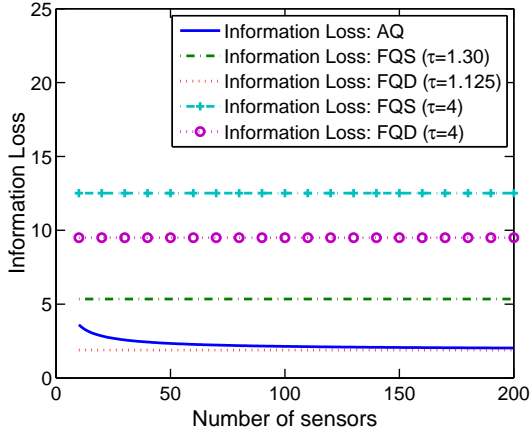


Fig. 7: Laplace observations: Information loss of the FQ and AQ schemes relative to the ML estimator using unquantized data,  $\sigma = 1$ .

show that the FQ schemes are able to achieve an estimation performance close to that of the clairvoyant estimator using unquantized data when the optimum quantization thresholds are employed. A drawback of the FQ approach is that its estimation performance is sensitive to the quantization threshold, whose choice is always tricky in practice since the optimum thresholds are dependent on the unknown parameter. The proposed AQ scheme, in contrast to the FQ approach, can effectively address this problem. Our analysis shows that the proposed AQ approach is asymptotically optimum. Without any prior knowledge of the unknown parameter, it yields an asymptotic CRB equivalent to that of the FQ approach with the optimum threshold. Simulation results were presented to corroborates our claims.

#### APPENDIX A PROOF OF PROPOSITION 1

By noting that

$$\frac{\partial F_V(\tau/\sigma)}{\partial \sigma} = -\frac{\tau}{\sigma^2} p_V(\tau/\sigma) \quad (32)$$

the first derivative and the second derivative of  $L_{\text{FQS}}(\sigma)$  are given as follows, respectively

$$\begin{aligned} \dot{L}_{\text{FQS}}(\sigma) &= \sum_{n=1}^N \left\{ b_n \frac{\partial \log(1 - F_V(\tau/\sigma))}{\partial \sigma} \right. \\ &\quad \left. + (1 - b_n) \frac{\partial \log F_V(\tau/\sigma)}{\partial \sigma} \right\} \\ &= \frac{\tau}{\sigma^2} \sum_{n=1}^N \left\{ b_n \frac{p_V(\tau/\sigma)}{1 - F_V(\tau/\sigma)} - (1 - b_n) \frac{p_V(\tau/\sigma)}{F_V(\tau/\sigma)} \right\} \\ &= \frac{\tau}{\sigma^2} \sum_{n=1}^N \frac{p_V(\tau/\sigma)(b_n - 1 + F_V(\tau/\sigma))}{F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))} \quad (33) \end{aligned}$$

$$\begin{aligned} \ddot{L}_{\text{FQS}}(\sigma) &= \frac{1}{\sigma^2} \sum_{n=1}^N \left\{ \frac{-2\frac{\tau}{\sigma} p_V(\tau/\sigma)(b_n - 1 + F_V(\tau/\sigma))}{F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))} \right. \\ &\quad \left. + \frac{-\frac{\tau^2}{\sigma^2} p_V^2(\tau/\sigma) + \tau(b_n - 1 + F_V(\tau/\sigma)) \dot{p}_V(\tau/\sigma)}{F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))} \right. \\ &\quad \left. - \frac{\frac{\tau^2}{\sigma^2} p_V^2(\tau/\sigma)(2F_V(\tau/\sigma) - 1)(b_n - 1 + F_V(\tau/\sigma))}{[F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))]^2} \right\} \quad (34) \end{aligned}$$

where  $\dot{p}_V(\tau/\sigma) \triangleq \frac{\partial p_V(\tau/\sigma)}{\partial \sigma}$ . By setting the first derivative of  $L_{\text{FQS}}(\sigma)$ ,  $\dot{L}_{\text{FQS}}(\sigma)$ , to zero, the ML estimate of  $\sigma$  can be easily obtained as (5).

The Fisher information for the estimation problem is given by

$$J_{\text{FQS}}(\sigma) = -E_{b_n}[\ddot{L}(\sigma)] \quad (35)$$

where  $E_{b_n}$  denotes the expectation w.r.t. the distribution  $P(b_n; \sigma)$  (see (3)). Since

$$\begin{aligned} E_{b_n}[(b_n - 1 + F_V(\tau/\sigma))] &= -F_V(\tau/\sigma)(1 - F_V(\tau/\sigma)) \\ &\quad + (1 - F_V(\tau/\sigma))F_V(\tau/\sigma) \\ &= 0 \quad (36) \end{aligned}$$

it can be readily verified that

$$J_{\text{FQS}}(\sigma) = N \frac{\tau^2}{\sigma^4} \frac{p_V^2(\tau/\sigma)}{F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))}. \quad (37)$$

Hence the CRB is given by

$$\begin{aligned} \text{CRB}_{\text{FQS}}(\sigma) &= \frac{1}{J(\sigma)} \\ &= \frac{1}{N} \frac{\sigma^4}{\tau^2} \frac{F_V(\tau/\sigma)(1 - F_V(\tau/\sigma))}{p_V^2(\tau/\sigma)}. \quad (38) \end{aligned}$$

The proof is completed here.

#### APPENDIX B PROOF OF PROPOSITION 2

The first derivative and the second derivative of  $L_{\text{FQD}}(\sigma)$  are given as follows respectively

$$\begin{aligned} \dot{L}_{\text{FQD}}(\sigma) &= \sum_{n=1}^N \left\{ b_n \frac{\partial \log[2 - 2F_V(\tau/\sigma)]}{\partial \sigma} \right. \\ &\quad \left. + (1 - b_n) \frac{\partial \log[2F_V(\tau/\sigma) - 1]}{\partial \sigma} \right\} \\ &= \frac{\tau}{\sigma^2} \sum_{n=1}^N \left\{ b_n \frac{p_V(\tau/\sigma)}{1 - F_V(\tau/\sigma)} - 2(1 - b_n) \frac{p_V(\tau/\sigma)}{2F_V(\tau/\sigma) - 1} \right\} \\ &= \frac{\tau}{\sigma^2} \sum_{n=1}^N \frac{p_V(\tau/\sigma)(b_n - 2 + 2F_V(\tau/\sigma))}{(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)} \quad (39) \end{aligned}$$

$$\begin{aligned} \ddot{L}_{\text{FQD}}(\sigma) &= \frac{1}{\sigma^2} \sum_{n=1}^N \left\{ \frac{-2\frac{\tau}{\sigma} p_V(\tau/\sigma)(b_n - 2 + 2F_V(\tau/\sigma))}{(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)} \right. \\ &\quad \left. + \frac{-2\frac{\tau^2}{\sigma^2} p_V^2(\tau/\sigma) + \tau(b_n - 2 + 2F_V(\tau/\sigma)) \dot{p}_V(\tau/\sigma)}{(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)} \right. \\ &\quad \left. - \frac{\frac{\tau^2}{\sigma^2} p_V^2(\tau/\sigma)(4F_V(\tau/\sigma) - 3)(b_n - 2 + 2F_V(\tau/\sigma))}{[(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)]^2} \right\}. \quad (40) \end{aligned}$$

Since

$$\begin{aligned} & E_{b_n}[(b_n - 2 + 2F_V(\tau/\sigma))] \\ &= -2(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1) \\ &\quad + 2(2F_V(\tau/\sigma) - 1)(1 - F_V(\tau/\sigma)) \\ &= 0 \end{aligned} \quad (41)$$

it can be readily verified that the Fisher information is

$$\begin{aligned} J_{\text{FQD}}(\sigma) &= -E_{b_n}[\ddot{L}_{\text{dual}}(\sigma)] \\ &= -2N \frac{\tau^2}{\sigma^4} \frac{p_V^2(\tau/\sigma)}{(1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)}. \end{aligned} \quad (42)$$

Therefore the CRB is given by

$$\text{CRB}_{\text{FQD}}(\sigma) = \frac{1}{2N} \frac{\sigma^4 (1 - F_V(\tau/\sigma))(2F_V(\tau/\sigma) - 1)}{\tau^2 p_V^2(\tau/\sigma)}. \quad (43)$$

The proof is completed here.

### APPENDIX C PROOF OF PROPOSITION 3

Note that sensor  $m$  computes its threshold as  $\tau_m = \mu \hat{\sigma}_m$ , where  $\hat{\sigma}_m$  is estimated as

$$\begin{aligned} \hat{\sigma}_m &= \arg \max_{\sigma} L_m(\sigma; \{b_k\}_{k=1}^{m-1}, \{\tau_k\}_{k=1}^{m-1}) \\ &= \arg \max_{\sigma} \sum_{k=1}^{m-1} \left[ b_k \log[2 - 2F_V(\tau_k/\sigma)] \right. \\ &\quad \left. + (1 - b_k) \log[2F_V(\tau_k/\sigma) - 1] \right]. \end{aligned} \quad (44)$$

It can be easily verified that the above log-likelihood function satisfies the ‘‘regularity’’ conditions and, hence, for large data records (i.e.  $m$  is large), the ML estimate  $\hat{\sigma}_m$  is consistent<sup>2</sup> [31], [32]. Consequently, for any small  $\epsilon > 0$ , we can find a sufficiently large  $m$  such that

$$P(|\hat{\sigma}_n - \sigma| < \epsilon) > 1 - \epsilon \quad n \geq m. \quad (45)$$

Considering (26), we express  $J_{\text{AQ}}(\sigma)$  as the summation of the following two terms

$$\begin{aligned} J_{\text{AQ}}(\sigma) &= \frac{2}{\sigma^2} \sum_{n=1}^{m-1} \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &\quad + \frac{2}{\sigma^2} \sum_{n=m}^N \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &\triangleq J_1 + J_2 \end{aligned} \quad (46)$$

<sup>2</sup>The consistency of the ML estimator has been studied and proved by some earlier works, e.g. [31, Appendix 7B] and the reference therein. In [31, Appendix 7B], the authors stated that the ML estimator is consistent if the likelihood function satisfies the following two ‘‘regularity’’ conditions: 1) The first-order and second-order derivatives of the log-likelihood function are well defined; 2)  $E[\dot{L}_{\text{AQ}}(\sigma)] = 0$ . These two conditions are exactly satisfied for our case. The second condition can be easily verified by utilizing the property (41). However, to simplify the discussion, [31, Appendix 7B] only provided a proof for i.i.d. observations. The consistency proof for the generally dependent observations can be found in [32]. Note that for the AQ scheme, the quantized data  $\{b_n\}$  is non i.i.d. due to the fact that the threshold  $\tau_n$  used to generate  $b_n$  is dependent on the previous data  $\{b_m\}_{m=1}^{n-1}$ .

where  $m$  is chosen to satisfy (45). Clearly, we have

$$\begin{aligned} 0 < J_1 &\triangleq \frac{2}{\sigma^2} \sum_{n=1}^{m-1} \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &< \frac{2}{\sigma^2} (m-1) G^{-1}(\tau_{\min}; \sigma). \end{aligned} \quad (47)$$

where  $\tau_{\min}$  is the value minimizing the function  $G(\tau; \sigma)$  defined in (27). By comparing (27) to (13), it is easy to see that  $\tau_{\min} = \tau_{\text{opt}} = \mu\sigma$ , where  $\tau_{\text{opt}}$  denotes the optimum quantization threshold for the FQD scheme.

Similarly,  $J_2$  is upper bounded by

$$\begin{aligned} J_2 &\triangleq \frac{2}{\sigma^2} \sum_{n=m}^N \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &< \frac{2}{\sigma^2} (N - m + 1) G^{-1}(\tau_{\min}; \sigma). \end{aligned} \quad (48)$$

On the other hand, its lower bound can be derived as

$$\begin{aligned} J_2 &= \frac{2}{\sigma^2} \sum_{n=m}^N \int P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &= \frac{2}{\sigma^2} \sum_{n=m}^N \left[ \int_{\tau_n: |\tau_n - \tau_{\min}| < \mu\epsilon} P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \right. \\ &\quad \left. + \int_{\tau_n: |\tau_n - \tau_{\min}| \geq \mu\epsilon} P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \right] \\ &> \frac{2}{\sigma^2} \sum_{n=m}^N \int_{\tau_n: |\tau_n - \tau_{\min}| < \mu\epsilon} P(\tau_n; \sigma) G^{-1}(\tau_n; \sigma) d\tau_n \\ &\stackrel{(a)}{>} \frac{2}{\sigma^2} (G^{-1}(\tau_{\min}; \sigma) - \delta) \sum_{n=m}^N \int_{\tau_n: |\tau_n - \tau_{\min}| < \mu\epsilon} P(\tau_n; \sigma) d\tau_n \\ &= \frac{2}{\sigma^2} (G^{-1}(\tau_{\min}; \sigma) - \delta) \sum_{n=m}^N P(\tau_n : |\tau_n - \tau_{\min}| < \mu\epsilon; \sigma) \\ &\stackrel{(b)}{>} \frac{2}{\sigma^2} (G^{-1}(\tau_{\min}; \sigma) - \delta) (N - m + 1) (1 - \epsilon) \end{aligned} \quad (49)$$

where (a) comes from the fact that  $G^{-1}(\tau)$  is a continuous function, therefore for any  $\delta > 0$ , we can find a  $\epsilon$  such that for all  $\tau : |\tau - \tau_{\min}| < \mu\epsilon$ , we have  $G^{-1}(\tau_{\min}; \sigma) - G^{-1}(\tau; \sigma) < \delta$ ; (b) follows from

$$\begin{aligned} P(\tau_n : |\tau_n - \tau_{\min}| < \mu\epsilon; \sigma) &= P(\hat{\sigma}_n : |\mu\hat{\sigma}_n - \mu\sigma| < \mu\epsilon; \sigma) \\ &= P(|\hat{\sigma}_n - \sigma| < \epsilon) \\ &> 1 - \epsilon \end{aligned} \quad (50)$$

in which the inequality comes from (45).

Combining (47)–(49), we therefore have

$$\begin{aligned} \frac{2NG^{-1}(\tau_{\min}; \sigma)}{\sigma^2} &> J_{\text{AQ}}(\sigma) > \\ &\frac{2(N - m + 1)(1 - \epsilon)(G^{-1}(\tau_{\min}; \sigma) - \delta)}{\sigma^2}. \end{aligned} \quad (51)$$

The CRB is, therefore, lower bounded and upper bounded by

$$\begin{aligned} \frac{\sigma^2 G^{-1}(\tau_{\min}; \sigma)}{2N} &< \text{CRB}_{\text{AQ}}(\sigma) < \\ &\frac{\sigma^2 G^{-1}(\tau_{\min}; \sigma)}{2N} \frac{NG^{-1}(\tau_{\min}; \sigma)}{(1 - \epsilon)(G^{-1}(\tau_{\min}; \sigma) - \delta)(N - m + 1)}. \end{aligned} \quad (52)$$

Considering  $N \gg m$ ,  $m$  is sufficiently large to ensure  $\epsilon \rightarrow 0$  and  $\delta \rightarrow 0$ , and noticing that  $\tau_{\min} = \tau_{\text{opt}}$ , hence we have

$$\text{NCRB}_{\text{AQ}}(\sigma) \rightarrow \text{NCRB}_{\text{FQD}}(\tau_{\text{opt}}; \sigma) = \frac{\sigma^2 G_{\text{FQD}}(\tau_{\text{opt}}; \sigma)}{2}. \quad (53)$$

The proof is completed here.

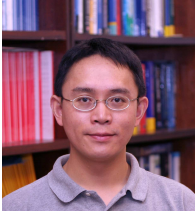
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