2b) Solve $\frac{dy}{dx} = y\sqrt{x+2}\,; \quad y(2) = 1$

The differential equation is separable,
and we may write:

$$\frac{dy}{y} = \sqrt{x+2}\,dx \qquad (1)$$

$$\int \frac{dy}{y} = \int \sqrt{x+2}\,dx$$

Integrate both sides:

$$\ln|y| = \tfrac{2}{3}(x+2)^{3/2} + C \qquad (2)$$

Take exponentials:

$$y(x) = \exp\left\{\tfrac{2}{3}(x+2)^{3/2} + C\right\} \qquad (3)$$

Finally, apply the boundary condition:

$$y(2) = \exp\left\{\tfrac{16}{3} + C\right\} = 1 \qquad (4)$$

which implies $C = -\frac{16}{3}$.

Therefore $y = \exp\left\{\tfrac{2}{3}(x+2)^{3/2} - \tfrac{16}{3}\right\}$