1a)

$$\frac{du}{dv} + \frac{\sec^2 v}{\tan v}u = \cot v \qquad u(\frac{\pi}{4}) = \frac{\pi}{2} \qquad \#$$

$$\#$$

$$\frac{du}{dv} + P(v)u = Q(v) \qquad \text{(Linear DE)} \qquad \#$$

$$P(v) = \frac{\sec^2 v}{\tan v}$$

$$\text{Integrating Factor} = I = e^{\int P(v)dv}$$

$$\#$$

$$I = e^{\int \frac{\sec^2 v}{\tan v} dv} = e^{\ln(\tan v)} = \tan v$$

$$\#$$

$$I\left(\frac{du}{dv} + \frac{\sec^2 v}{\tan v}u\right) = I\cot v \qquad (1) \qquad \#$$

$$\frac{du}{dv}\tan v + u\sec^2 v = 1 \qquad (2)$$

We may rewrite the DE as $\#$

$$\frac{d}{dv}(u\tan v) = 1$$

$$\frac{d}{dv}(u\tan v) = 1$$

$$\int d(u\tan v) = \int dv$$

$$u\tan v = v + C \qquad (3) \qquad \#$$

$$u(v) = (v + C)\cot v \qquad (4)$$

$$u(\frac{\pi}{4}) = \frac{\pi}{4} + C = \frac{\pi}{2}$$

$$\text{which implies } C = \frac{\pi}{4}$$

$$\#$$

$$\text{Thus, } u(v) = \left(v + \frac{\pi}{4}\right)\cot v$$