

2a) Find the general solution of

$$y'' - 10y' + 34y = 0$$

in terms of real functions.

Assuming a solution of the form $y = e^{rx}$ and substituting into the DE yields

$$(r^2 - 10r + 34)e^{rx} = 0 \quad (1)$$

$$r^2 - 10r + 34 = 0 \quad (2)$$

which has solutions: $r = 5 - 3i, 5 + 3i$

Then $y = e^{(5-3i)x}$ and $y = e^{(5+3i)x}$ are two linearly independent solutions to the DE.

Then the linearly independent set of solutions

$$y = e^{(5-3i)x} \text{ and } y = e^{(5+3i)x}$$

may be replaced by the linearly independent set

$$y = e^{5x} \cos 3x \text{ and } y = e^{5x} \sin 3x$$

Thus $y = c_1 e^{5x} \cos 3x + c_2 e^{5x} \sin 3x \quad (3)$