

3a) Find the Laplace transform of  $g(x) = x^2 \sin x$  using the formula  $\mathcal{L}\{(-x)^n f(x)\} = \frac{d^n}{ds^n} \hat{f}(s)$  (1)

$$\mathcal{L}\{x^2 \sin x\} = \mathcal{L}\{(-x)^2 \sin x\} \quad (2)$$

Thus we can apply formula (1), with  $f(x) = \sin x$  and  $n = 2$ .

From memory or from a table,  $\mathcal{L}\{\sin x\} = \frac{1}{s^2+1}$

$$\text{Therefore } \mathcal{L}\{(-x)^2 \sin x\} = \frac{d^2}{ds^2} \left( \frac{1}{s^2+1} \right) \quad (3)$$

$$\frac{d^2}{ds^2} \left( \frac{1}{s^2+1} \right) = \frac{d}{ds} \left( \frac{-2s}{(s^2+1)^2} \right) = \frac{6s^2-2}{(s^2+1)^3} \quad (4)$$

Thus the Laplace Transform of  $x^2 \sin x$  is

$$\frac{6s^2 - 2}{(s^2 + 1)^3}$$