1a) Solve $y' + 2xy = 4x$ $y(0) = 4$

The DE is linear: $P(x) = 2x,$ $Q(x) = 4x$

The integrating factor is $I = e^{\int P dx} = e^{x^2}$

Multiplying both sides by I yields:

$\frac{d}{dx}\left[y e^{x^2} \right] = 4x e^{x^2}$ (1)

$y e^{x^2} = \int 4x e^{x^2} dx = 2 e^{x^2} + C$ (2)

Therefore $y = C e^{-x^2} + 2$ (3)

Now, applying the initial condition to the
general solution $y = C e^{-x^2} + 2,$ we get:

$y(0) = 4 \Rightarrow 4 = C + 2 \Rightarrow C = 2$ (4)

So finally, $y = 2\left(e^{-x^2} + 1 \right)$