1c) Solve the first-order differential equation:

$$(ye^{xy} + 2xy)dx + (xe^{xy} + x^2)dy = 0$$

Notice the DE is neither linear nor separable.

Check for exactness:

$$M = ye^{xy} + 2xy, \qquad N = xe^{xy} + x^2$$

The equation is exact, since:

$$M_y = N_x = xye^{xy} + e^{xy} + 2x \qquad (1)$$

Then the equation has the form $df = 0$,
and consequently $\frac{\partial f}{\partial x} = M$

$$f = \int M dx + g(y) = \int (ye^{xy} + 2xy)dx + g(y) \qquad (2)$$

So $f = e^{xy} + x^2 y + g(y)$ where $g(y)$ is an unknown function of y

Now to find $g(y)$, note that $\frac{\partial f}{\partial y} = N$, that is:

$$xe^{xy} + x^2 + g'(y) = xe^{xy} + x^2 \qquad (3)$$

So $g'(y) = 0$, and therefore $g(y) = C_1$

Integrating the exact differential $df = 0$

we get $f = e^{xy} + x^2 y + C_1 = C_2 \qquad (4)$

or $e^{xy} + x^2y + C = 0$; where $C = C_1 - C_2$