Exam II Problems for Ma 221 2004 Fall.

1 Exam IIA

1. Consider the differential equation

$$y'' - 3y' - 4y = 3e^{2x} + 2\sin x - 8e^{-x}$$

- 1 a (10 pts.) Find the homogeneous solution of this equation.
- 1 b (25 pts.) Find a particular solution of this equation.
- 1 c (10 pts.) Give a general solution of this equation.
- 2 Consider the differential equation

$$x^{2}y'' - 4xy' + 6y = x^{2}\ln x \quad x > 0 \tag{(*)}$$

- 2 a (10 pts.) Find two linearly independent solutions of the homogeneous equation corresponding to (*).
- 2 b (10 pts.) Find the value of the Wronskian of the two linearly independent solutions you found in 2a.
- 2c (25 pts.) Find a particular solution to (*).

Note:

$$\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1} + C$$

$$\int \frac{\ln x}{x^n} dx = -\left[\frac{\ln x}{(n-1)x^{n-1}} + \frac{1}{(n-1)^2x^{n-1}}\right] + C \qquad n \neq 0, 1$$

2 d (10 pts.) Give a general solution to (*).

2 Exam IIB

1. Consider the differential equation

$$y'' + 4y' - 5y = 3e^{2x} + 2\cos x - 8e^x$$

- 1 a (10 pts.) Find the homogeneous solution of this equation.
- 1 b (25 pts.) Find a particular solution of this equation.
- 1 c (10 pts.) Give a general solution of this equation.
- 2 Consider the differential equation

$$x^{2}y'' - 3xy' + 3y = x^{3}\ln x \quad x > 0 \tag{(*)}$$

- 2 a (10 pts.) Find two linearly independent solutions of the homogeneous equation corresponding to (*).
- 2 b (10 pts.) Find the value of the Wronskian of the two linearly independent solutions you found in 2a.
- 2c (25 pts.) Find a particular solution to (*).

Note:

$$\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1} + C$$
$$\int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C$$

2 d (10 pts.) Give a general solution to (*).