Name:_____ Lecturer _____

ID#:_____ Lecture Section: _____

Ma 221 Exam IA Solutions
05F

I pledge my honor that I have abided by the Stevens Honor System._____

You may not use a calculator, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

In Problems 1 – 3 solve the equations:

1 [25 **pts**.]

$$\frac{dy}{dx} = \frac{2xy}{1+y} \quad y(0) = 1$$

Solution: This equation is separable when we write it as

$$\frac{1+y}{y}dy = 2xdx$$

or

$$\left(\frac{1}{y} + 1\right)dy = 2xdx$$

Integrating yields

$$\ln y + y = x^2 + C$$

The initial condition implies

$$\ln 1 + 1 = C$$

so the solution is given by

$$\ln y + y = x^2 + 1$$

2 [25 **pts**.]

$$\left(y - \frac{2}{x}\right)dx = (y-x)dy$$

Solution: We rewrite the equation as

$$\left(y - \frac{2}{x}\right)dx + (x-y)dy = 0$$

Here $M = \left(y - \frac{2}{x}\right)$ and $N = x - y$. Then $M_y = 1 = N_x$ and the equation is exact. Hence there exists a function $F(x,y)$ such that

$$F_x = M = y - \frac{2}{x}$$
$$F_y = N = x - y$$

Integrating with respect to x we have

$$F(x,y) = xy - 2\ln x + g(y)$$

Then

$$F_y = x + g'(y) = N = x - y$$

so $g(y) = -\frac{y^2}{2} + C$. Thus

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$$F(x,y) = xy - 2\ln x - \frac{y^2}{2} + C$$

and the solution is given by

$$xy - 2\ln x - \frac{y^2}{2} = K$$

3 [25 **points**]

$$xy' + y = x^2y^2 \quad y(1) = 1$$

Solution: We rewrite the equation as

$$y' + \tfrac{1}{x}y = xy^2$$

and see that it is a Bernoulli equation. Multiplying by y^{-2} we have

$$y^{-2}y' + \tfrac{1}{x}y^{-1} = x$$

Letting $z = y^{-1}$ means that $z' = -y^{-2}y'$ and the DE can be written as

$$-z' + \tfrac{1}{x}z = x$$

or

$$z' - \tfrac{1}{x}z = -x$$

This is now a first order linear DE in z with $P = -\frac{1}{x}$. Thus $e^{\int P dx} = e^{-\ln x} = \frac{1}{x}$ we multiply the DE by $\frac{1}{x}$. We get

$$\left(\tfrac{1}{x}\right)z' - \left(\tfrac{1}{x^2}\right)x = -1$$

or

$$\frac{d}{dx}\left(\tfrac{z}{x}\right) = -1$$

Thus

$$\tfrac{z}{x} = -x + C$$

so

$$z = \frac{1}{y} = -x^2 + Cx$$

The initial condition $y(.5) = .5$ implies that

$$2 = -.25 + C(.5)$$
$$C = 2(2.25) = 4.5$$

so $C = .75$ and the solution is given by

$$\frac{1}{y} = -x^2 + 4.5x$$

SNB check: $\begin{array}{c} x\frac{dy}{dx} + y = x^2y^2 \\ y(.5) = .5 \end{array}$, Exact solution is: $\left\{\frac{1}{4.5x - x^2}\right\}$,

4a [10 **pts.**] The function $y(t) = Ae^{-4t}$ is a solution of the equation

$$y'' + 5y' + 2y = 10e^{-4t}$$

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Find A.

Solution:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = 16Ae^{-4t} - 20Ae^{-4t} + 2Ae^{-4t} = -2Ae^{-t} = 10e^{-t}$$

so $A = -5$.

4b $\left[15\ \textbf{pts.}\right]$ Solve the equation

$$y' = y + e^{-t}$$

Solution: Write the equation as

$$\frac{dy}{dt} - y = e^{-t}$$

Then, since the equation is first order linear $e^{\int P dt} = e^{\int -dt} = e^{-t}$ and the equation may be rewritten as

$$(e^{-t}y)' = e^{-2t}$$

so

$$e^{-t}y = -\frac{1}{2}e^{-2t} + C$$

Thus

$$y = -\frac{1}{2}e^{-t} + Ce^{t}$$