

Name: _____

Lecturer _____

ID#: _____

Lecture Section: _____

Ma 221 05F

Exam IB Solutions

I pledge my honor that I have abided by the Stevens Honor

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

In Problems 1 – 3 solve the equations:

1 [25 pts.]

$$\frac{dy}{dx} = \frac{1-x}{2xy} \quad y(1) = 0$$

Solution: This equation is separable when we write it as

$$2ydy = \frac{1-x}{x} dx$$

or

$$2ydy = \left(\frac{1}{x} - 1 \right) dx$$

Integrating yields

$$y^2 = \ln x - x + C$$

The initial condition implies

$$0 = \ln 1 - 1 + C$$

$$\Rightarrow C = 1$$

so the solution is given by

$$y^2 = \ln x - x + 1$$

2. [25 pts.]

$$\left(\frac{5}{x} + 3y \right) dx = (2y - 3x) dy$$

Solution: We rewrite the equation as

$$\left(\frac{5}{x} + 3y \right) dx + (3x - 2y) dy = 0$$

Here $M = \left(\frac{5}{x} + 3y \right)$ and $N = 3x - 2y$. Then $M_y = 3 = N_x$ and the equation is exact. Hence there exists a function $F(x, y)$ such that

$$F_x = M = \frac{5}{x} + 3y$$

$$F_y = N = 3x - 2y$$

Integrating with respect to x we have

$$F(x, y) = 5 \ln x + 3yx + g(y)$$

Then

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$$F_y = 3x + g'(y) = N = 3x - 2y$$

so $g(y) = -y^2 + C$. Thus

$$F(x, y) = 5 \ln x + 3yx - y^2 + C$$

and the solution is given by

$$5 \ln x + 3yx - y^2 = K$$

3 [25 points]

$$xy' - y = x^2y^4 \quad y(1) = 1$$

Solution: We rewrite the equation as

$$y' - \frac{1}{x}y = xy^4$$

and see that it is a Bernoulli equation. Multiplying by y^{-4} we have

$$y^{-4}y' - \frac{1}{x}y^{-3} = x$$

Letting $z = y^{-3}$ means that $z' = -3y^{-4}y'$ and the DE can be written as

$$-\frac{1}{3}z' - \frac{1}{x}z = x$$

or

$$z' + \frac{3}{x}z = -3x$$

This is now a first order linear DE in z with $P = \frac{3}{x}$. Thus $e^{\int P dx} = e^{3 \ln x} = x^3$ we multiply the DE by x^3 . We get

$$x^3z' + 3x^2z = -3x^4$$

or

$$\frac{d}{dx}(zx^3) = -3x^4$$

Thus

$$zx^3 = -\frac{3}{5}x^5 + C$$

so

$$z = \frac{1}{y^3} = -\frac{3}{5}x^2 + Cx^{-3}$$

The initial condition $y(1) = 1$ implies that

$$1 = -\frac{3}{5} + C$$

so $C = \frac{8}{5}$ and the solution is given by

$$\frac{1}{y^3} = -\frac{3}{5}x^2 + \frac{8}{5}x^{-3}$$

4a [10 pts.] The function $y(t) = Ae^{-6t}$ is a solution of the equation

$$y'' + 3y' + 4y = 44e^{-6t}$$

Find A.

Solution:

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$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4y = 36Ae^{-6t} - 18Ae^{-6t} + 4Ae^{-6t} = 22Ae^{-6t} = 44e^{-6t}$$

so $A = 2$.

4b [15 pts.] Solve the equation

$$y' = 2y + e^{3t}$$

Solution: Write the equation as

$$\frac{dy}{dt} - 2y = e^{3t}$$

Then, since the equation is first order linear $e^{\int P dt} = e^{\int -2 dt} = e^{-2t}$ and the equation may be rewritten as

$$(e^{-2t}y)' = e^t$$

so

$$e^{-2t}y = e^t + C$$

Thus

$$y = e^{3t} + Ce^{2t}$$