

Name: _____

Lecturer _____

Lecture Section: _____

Ma 221 07S

Exam IA Solutions

I pledge my honor that I have abided by the Stevens Honor

System. _____

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 _____

#2 _____

#3 _____

#4 _____

Total Score _____

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Solve the equations:

1 [25 pts.]

$$e^y dx + (xe^y + 2y)dy = 0$$

Solution: Here $M = e^y$ and $N = xe^y + 2y$, so $M_y = N_x = e^y$ and this equation is exact. Thus there exists $f(x,y)$ such that

$$f_x = M = e^y$$

$$f_y = N = xe^y + 2y$$

Therefore

$$f(x,y) = xe^y + g(y)$$

so

$$f_y = xe^y + g'(y) = xe^y + 2y$$

Therefore

$$g'(y) = 2y \Rightarrow g(y) = y^2 + C$$

and

$$f(x,y) = xe^y + g(y) = xe^y + y^2 + C$$

so the solution is

$$xe^y + y^2 = K$$

2 [25 pts.]

$$y' + \frac{1}{x}y = \frac{2}{x} \quad y(1) = 1$$

Solution: This is first order linear. $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$. Mutliplying the equation by x , we get

$$xy' + y = \frac{d}{dx}(xy) = 2$$

Then

$$xy = 2x + c$$

or

$$y = 2 + \frac{c}{x}$$

The IC implies

$$1 = 2 + c$$

so $c = -1$ and

$$y = 2 - \frac{1}{x}$$

SNB check:

$$y' + \frac{1}{x}y = \frac{2}{x}$$

$$y(1) = 1$$

, Exact solution is: $\left\{ \frac{1}{x}(2x - 1) \right\}$

.

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3 [25 pts.]

$$xy^2 - y'x^2 = 0 \quad y(1) = 2$$

Solution:

$$x^2 \frac{dy}{dx} = xy^2$$

or

$$\frac{dy}{y^2} = \frac{dx}{x}$$

Integrating we get

$$-\frac{1}{y} = \ln x + c$$

The initial condition implied

$$-\frac{1}{2} = c$$

so

$$-\frac{1}{y} = \ln x - \frac{1}{2}$$

4b [25 pts.]

$$y' + xy = xy^4$$

Solution: This is a Bernoulli equation.

$$y^{-4}y' + xy^{-3} = x$$

Let $z = y^{-3}$ so that $z' = -3y^{-4}y'$. The DE becomes

$$\frac{z'}{-3} + xz = x$$

or

$$z' - 3xz = -3x$$

Then $e^{\int -3x dx} = e^{-\frac{3}{2}x^2}$. Multiply the DE by this to get

$$e^{-\frac{3}{2}x^2} z' - 3xe^{-\frac{3}{2}x^2} z = -3xe^{-\frac{3}{2}x^2}$$

or

$$\frac{d}{dx} \left(e^{-\frac{3}{2}x^2} z \right) = -3xe^{-\frac{3}{2}x^2}$$

Integrating we get

$$e^{-\frac{3}{2}x^2} z = e^{-\frac{3}{2}x^2} + c$$

and therefore

$$y^{-3} = 1 + ce^{\frac{3}{2}x^2}$$