Name:____________________________________ Lecturer _________________________

Lecture Section: ___________

Ma 221 Exam IB Solutions
07S

I pledge my honor that I have abided by the Stevens Honor System.____

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

Total Score ________

Name:____________________________________ Lecturer __________________________

Lecture Section: ____________

Solve the equations:

1 $\left[25 \textbf{ pts.}\right]$

$$e^y dx + (xe^y + 4y)dy = 0$$

Solution: Here $M = e^y$ and $N = xe^y + 4y$, so $M_y = N_x = e^y$ and this equation is exact. Thus there exists $f(x,y)$ such that

$$f_x = M = e^y$$
$$f_y = N = xe^y + 4y$$

Therefore

$$f(x,y) = xe^y + g(y)$$

so

$$f_y = xe^y + g'(y) = N = xe^y + 4y$$

Therefore

$$g'(y) = 4y \Rightarrow g(y) = 2y^2 + C$$

and

$$f(x,y) = xe^y + g(y) = xe^y + 2y^2 + C$$

so the solution is

$$xe^y + 2y^2 = K$$

2 $\left[25 \textbf{ pts.}\right]$

$$y' + \frac{1}{x}y = \frac{4}{x}; \quad y(1) = 1$$

Solution: This is first order linear.

Integrating Factor $I(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$. Mutliplying the equation by x, we get

$$xy' + y = \frac{d}{dx}(xy) = 4$$

Then

$$xy = 4x + c$$

or

$$y = 4 + \frac{c}{x}$$

The IC implies

$$1 = 4 + c$$

so $c = -3$ and

$$y = 4 - \frac{3}{x}$$

SNB check:

$$y' + \frac{1}{x}y = \frac{4}{x}$$
$$y(1) = 1$$

Name:___________________________________ Lecturer _________________________

Lecture Section: ___________

, Exact solution is: $\left\{\frac{1}{x}(4x-3)\right\}$
.

3 [**25 pts**.]

$$xy^2 - y'x^2 = 0; \;\; y(e) = 1$$

Solution:

$$x^2\frac{dy}{dx} = xy^2$$

or

$$\frac{dy}{y^2} = \frac{dx}{x}$$

Integrating we get

$$-\frac{1}{y} = \ln x + c$$

The initial condition implied

$$-2 = c$$

so

$$-\frac{1}{y} = \ln x - 2$$

4b $\left[25\right.$ **pts**.$\left.\right]$

$$y' + xy = xy^3$$

Solution: This is a Bernoulli equation.

$$y^{-3}y' + xy^{-2} = x$$

Let $z = y^{-2}$ so that $z' = -2y^{-3}y'$. The DE becomes

$$\frac{z'}{-2} + xz = x$$

or

$$z' - 2xz = -2x$$

Then $e^{\int -2xdx} = e^{-x^2}$. Multiply the DE by this to get

$$e^{-x^2}z' - 2xe^{-x^2}z = -2xe^{-x^2}$$

or

$$\frac{d}{dx}\left(e^{-x^2}z\right) = -2xe^{-x^2}$$

Integrating we get

$$e^{-x^2}z = e^{-x^2} + c$$

and therefore

$$y^{-2} = 1 + ce^{x^2}$$

or

$$y = 0$$

a trivial solution