Name:___________________________________ Lecturer _________________________

Lecture Section: ___________

Ma 221 08S Exam IB Solutions

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

Total Score ________

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Solve:

1 [25 **pts**.]

$$\left(3x^2y^2 - 2y^3\right)dx + \left(2x^3y - 6xy^2 + 3y\right)dy = 0$$

Solution: $M(x,y) = 3x^2y^2 - 2y^3$ and $N(x,y) = 2x^3y - 6xy^2 + 3y$. Therefore

$$\frac{\partial M}{\partial y} = 6x^2y - 6y^2 = \frac{\partial N}{\partial x}$$

and the DE is exact. Hence there exists $f(x,y)$ such that $f_x = M$ and $f_y = N$.

$$f_x = 3x^2y^2 - 2y^3 \Rightarrow f(x,y) = x^3y^2 - 2xy^3 + g(y)$$

Then

$$f_y = 2x^3y - 6xy^2 + g'(y) = N = 2x^3y - 6xy^2 + 3y$$

so

$$g'(y) = 3y \Rightarrow g(y) = \frac{3}{2}y^2 + C$$

$$f(x,y) = x^3y^2 - 2xy^3 + \frac{3}{2}y^2 + C$$

and the solution is given by

$$x^3y^2 - 2xy^3 + \frac{3}{2}y^2 = K$$

where K is and arbitrary constant.

2 [**25 pts**.]

$$(x-2)y' - y = 3 \quad y(0) = 1$$

Solution 1: Rewrite the equation as

$$y' - \frac{1}{x-2}y = \frac{3}{x-2}$$

Then since $P = -\frac{1}{x-2}$,

$$e^{\int P dx} = e^{-\int \frac{1}{x-2}dx} = e^{-\ln(x-2)} = \frac{1}{x-2}$$

Multiplying the DE by $\frac{1}{x-2}$ gives

$$\left(\frac{1}{x-2}\right)y' - \left(\frac{1}{x-2}\right)^2 y = \frac{3}{(x-2)^2}$$

or

$$\frac{d}{dx}\left(\frac{y}{x-2}\right) = \frac{3}{(x-2)^2}$$

Integrating yields

$$\frac{y}{x-2} = \frac{-3}{(x-2)} + c$$

or

$$y(x) = -3 + c(x-2) \qquad (*)$$

The initial condition gives

$$y(0) = 1 = -3 - 2c \Rightarrow -2$$

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so

$$y(x) = -3 - 2(x-2) = -2x + 1$$

Solution 2: The equation is separable and may be rewritten as

$$\frac{dy}{y+3} = \frac{dx}{x+1}$$

Integrating we get

$$\ln(y+3) = \ln(x+1) + K$$

or

$$\ln\left(\frac{y+3}{x+1}\right) = K$$

so

$$y + 3 = c(x+1)$$

which is equation $(*)$ above.

3 [25 **points**]

$$y' = \frac{6x^2 - 8x + 1}{4y+1} \quad y(1) = 2$$

Solution: We may write the DE as

$$(4y+1)dy = \left(6x^2 - 8x + 1\right)dx$$

Therefore the original equation is separable. Integrating we get

$$2y^2 + y = 2x^3 - 4x^2 + x + c$$

The initial condition implies

$$2(2)^2 + 2 = 2 - 4 + 1 + c$$

so $c = 11$ and the solution is

$$2y^2 + y = 2x^3 - 4x^2 + x + 11$$

4 $\left[\right.$25 **pts**.$\left.\right]$

$$y' = 2y + e^{-4x}y^{-3}$$

Solution: We rewrite the equation as

$$y' - 2y = e^{-4x}y^{-3}$$

so we are dealing with a Bernoulli equation. Multiplying by y^3 we get

$$y^3y' - 2y^4 = e^{-4x}$$

Let $z = y^4$ so $z' = 4y^3y'$ and we may rewrite the DE as

$$\frac{z'}{4} - 2z = e^{-4x}$$

or

$$z' - 8z = 4e^{-4x}$$

This is a first order linear equation in z with $P = -8.$ Thus we multiply the equation by $e^{\int Pdx} = e^{-\int 8dx} = e^{-8x}.$ Doing this leads to

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$$\frac{d}{dx}\left(e^{-8x}z\right) = 4e^{-12x}$$

Thus integration yields

$$e^{-8x}z = -\frac{4}{12}e^{-12x} + c$$

or

$$y^4 = z = -\frac{1}{3}e^{-4x} + ce^{8x}$$