Name:_________________________________ Lecturer ________________________

Lecture Section: ___________

# Ma 221 09S Exam IB Solutions

I pledge my honor that I have abided by the Stevens Honor System.______________________________________

**You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.**

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

Total Score ________

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Solve:

**1** [25 **pts**.]

$$\left(ye^{xy} - \frac{2}{y}\right)dx + \left(xe^{xy} + \frac{2x}{y^2} + y^3\right)dy = 0, \;\; y(1) = 1$$

Solution: Let

$$M = ye^{xy} - \frac{2}{y}$$

$$N = xe^{xy} + \frac{2x}{y^2} + y^3$$

Then

$$M_y = e^{xy} + xye^{xy} + \frac{2}{y^2} \quad \text{and} \;\; N_x = e^{xy} + xye^{xy} + \frac{2}{y^2}$$

and the equation is exact. Hence there exists $f(x,y)$ such that

$$f_x = M = ye^{xy} - \frac{2}{y} \;\; \text{and} \;\; f_y = N = xe^{xy} + \frac{2x}{y^2} + y^3$$

Integrating the expression for $f_x$ with respect to $x$ while holding $y$ fixed yields

$$f = e^{xy} - \frac{2x}{y} + g(y)$$

Then

$$f_y = xe^{xy} + \frac{2x}{y^2} + g'(y) = N = xe^{xy} + \frac{2x}{y^2} + y^3$$

Thus $g'(y) = y^3$ and $g(y) = \frac{y^4}{4} + c,$ where $c$ is a constant. Hence the solution is

$$f = e^{xy} - \frac{2x}{y} + \frac{y^4}{4} = k$$

We use the initial condition $y(1) = 1$ to find $k$.

$$e^1 - \frac{2(1)}{1} + \frac{1}{4} = k$$

Hence the solution is

$$e^{xy} - \frac{2x}{y} + \frac{y^4}{4} = e - \frac{7}{4}$$

**2** [25 **pts**.]

$$ty' + 3y = t^2 - t + 2 \quad y(1) = \frac{2}{3}$$

Solution: This equation is first order linear, so we write it as

$$y' + \frac{3}{t}y = t - 1 + \frac{2}{t}$$

The integrating factor is $e^{\int P(t)dt} = e^{\int \frac{3}{t}dt} = e^{3\ln t} = t^3.$ Multiplying the equation above by $t^3$ leads to

$$t^3y' + 3t^2y = t^4 - t^3 + 2t^2$$

or

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$$\frac{d\left(t^3 y\right)}{dt} = t^4 - t^3 + 2t^2$$

Integrating yields

$$t^3 y = \frac{t^5}{5} - \frac{t^4}{4} + \frac{2t^3}{3} + C$$

so

$$y = \frac{t^2}{5} - \frac{t}{4} + \frac{2}{3} + \frac{C}{t^3}$$

The initial condition implies

$$\frac{2}{3} = \frac{1}{5} - \frac{1}{4} + \frac{2}{3} + C$$

so $C = -\frac{1}{20}$ and the solution is

$$y = \frac{t^2}{5} - \frac{t}{4} + \frac{2}{3} - \frac{1}{20t^3}$$

.

**3** [25 **points**]

$$y' + 4y^5 = 6x^2y^5 \quad y(0) = 1$$

Solution: We may rewrite this equation as

$$\frac{dy}{dx} = y^5\left(6x^2 - 4\right)$$

or

$$\frac{dy}{y^5} = \left(6x^2 - 4\right)dx$$

so the equation is separable. Integrating yields

$$-\frac{y^{-4}}{4} = 2x^3 - 4x + C$$

The initial condition $y(0) = 1$ implies

$$-\frac{1}{4} = C$$

so the solution is

$$-\frac{y^{-4}}{4} = 2x^3 - 4x - \frac{1}{4}$$

or

$$\frac{y^{-4}}{4} = -2x^3 + 4x + \frac{1}{4}$$

**4** $\left[\right.$25 **pts**.$\left.\right]$

$$y' + xy = xe^{-2x^2}y^{-5}$$

This is a Bernoulli equation. Multiply both sides by $y^5$ to get

$$y^5y' + xy^6 = xe^{-2x^2}$$

Let $z = y^6$ so that $z' = 6y^5y'$. The DE may then be written as

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$$\frac{z'}{6} + xz = xe^{-2x^2}$$

or

$$z' + 6xz = 6xe^{-2x^2}$$

This equation is a first order linear DE in $z$. Then $I = e^{\int P dx} = e^{\int 6x dx} = e^{3x^2}$. Multiplying the DE by this integrating factor yields

$$z'e^{3x^2} + 6xe^{3x^2} = 6xe^{x^2}$$

or

$$\frac{d\left(ze^{3x^2}\right)}{dx} = 6xe^{x^2}$$

Integrating we have

$$ze^{3x^2} = 3e^{x^2} + C$$

Since $z = y^6$ the solution is

$$y^6 = 3e^{-2x^2} + Ce^{-3x^2}$$