$\qquad$

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem \#1 $\qquad$
$\qquad$
\#3 $\qquad$
\#4 $\qquad$

Total Score
Note: A table of Laplace Transforms is given at the end of the exam.
$\qquad$

1 (25 pts.) Use Laplace Transforms to solve

$$
y^{\prime \prime}+10 y^{\prime}+9 y=8 t \quad y(0)=-1 \quad y^{\prime}(0)=1
$$

$\qquad$

2a ( $\mathbf{1 5} \mathbf{~ p t s}$.) Use the definition of the Laplace transform to find $\mathscr{L}\{f(t)\}$ where

$$
f(t)=\left\{\begin{array}{c}
6 \quad 0 \leq t \leq 6 \\
t \quad t \geq 6
\end{array}\right.
$$

2b (10 pts.) Find

$$
\mathscr{L}^{-1}\left\{\frac{1+3 s}{s^{2}-8 s+21}\right\}
$$

$\qquad$

3 ( 25 pts.) Find the first six non-zero terms in the series solution near $x=0$ of the equation

$$
y^{\prime \prime}+x y=0
$$

Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first six nonzero terms of the solution.
$\qquad$

4 (25 pts.) Find the eigenvalues and eigenfunctions for

$$
y^{\prime \prime}+\lambda y=0 ; \quad y(0)=0, \quad y(3 \pi)=0
$$

Be sure to consider all values of $\lambda$.
$\qquad$

## Table of Laplace Transforms

| $f(t)$ | $F(s)=\mathscr{L}\{f\}(s)$ |  |  |
| :--- | :--- | :--- | :--- |
| $\frac{t^{n-1}}{(n-1)!}$ | $\frac{1}{s^{n}}$ | $n \geq 1$ | $s>0$ |
| $e^{a t}$ | $\frac{1}{s-a}$ |  | $s>a$ |
| $\sin b t$ | $\frac{b}{s^{2}+b^{2}}$ |  | $s>0$ |
| $\cos b t$ | $\frac{s}{s^{2}+b^{2}}$ |  | $s>0$ |
| $e^{a t f} f(t)$ | $\mathscr{L}\{f\}(s-a)$ |  |  |
| $t^{n} f(t)$ | $(-1)^{n} \frac{d^{n}}{d s^{n}}(\mathscr{L}\{f\}(s))$ |  |  |

