Name:___________________________________ Lecure Section ___

Ma 221 Exam IIIA 10S

I pledge my honor that I have abided by the Stevens Honor System.

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

Total Score ________

Note: A table of Laplace Transforms is given at the end of the exam.

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1a (13 **pts**.) Use Laplace Transforms to show that the solution $y(t)$ of the initial value problem

$$y'' + 3y' + 2y = 6e^{-t} \quad y(0) = 1 \;\; y'(0) = 2$$

has the Laplace Transform

$$\mathcal{L}\{y\} = \frac{s^2 + 6s + 11}{(s+1)^2(s+2)}$$

(**1b** 12 **pts**) Given that

$$\frac{s^2 + 6s + 11}{(s+1)^2(s+2)} = \frac{6}{(s+1)^2} - \frac{2}{s+1} + \frac{3}{s+2}$$

find $y(t)$.

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2a (15 pts.) Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} 0 & 0 \le t < 1 \\ 1 & 1 \le t \le 2 \\ 0 & t \ge 2 \end{cases}$$

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2b (15 **pts.**) Find

$$\mathcal{L}^{-1}\left\{ \frac{4s^3 - s^2 + 8s + 4}{s^2(s^2+4)} \right\}$$

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3 (25 **pts**.) Find the series solution near $x = 0$ of the equation

$$y'' - x^2 y = 0$$

Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first *six* nonzero terms of the solution.

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4 (20 **pts**.) Find the eigenvalues, λ, and eigenfunctions for

$$y'' + (\lambda + 1)y = 0; \quad y'(0) = 0, \quad y'(1) = 0$$

Be sure to consider all values of λ.

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		