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Ma 221 12S — Exam IB Solutions

Solve:

1 [25 **pts**.]

$$(2e^t x + 3)dt + (2e^t - 3)dx = 0 \quad x(1) = 1$$

Solution:

$$M = 2e^t x + 3 \quad \text{and} \quad N = 2e^t - 3$$

Thus

$$M_x = 2e^t = N_t$$

and the equation is exact. Thus there exists $f(t,x)$ such that

$$f_t = M \quad \text{and} \quad f_x = N$$

From the first equation we get by integrating with respect to t while holding x fixed

$$f = 2e^t x + 3t + g(x)$$

But

$$f_x = 2e^t + g'(x) = N = 2e^t - 3$$

Hence

$$g'(x) = -3$$

and $g(x) = -x + c$ so

$$f = 2e^t x + 3t - 3x + c$$

and the solution is given by

$$2e^t x + 3t - 3x = k$$

The initial condition implies

$$2e + 3 - 3 = k$$

so the solution is given by

$$2e^t x + 3t - 3x = 2e$$

or

$$x = \frac{2e - 3t}{2e^t - 3}$$

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2 [25 **pts**.]

$$\frac{dy}{dx} + 3y - e^x = 0 \quad y(0) = \frac{5}{4}$$

Solution: This equation is first order linear and we rewrite it as

$$\frac{dy}{dx} + 3y = e^x$$

Since $P(x) = 3$ the integrating factor is

$$e^{\int 3dx} = e^{3x}$$

Multiplying both sides of the DE by this we get

$$e^{3x}y' + 3e^{3x}y = e^{4x}$$

or

$$\frac{d\left(e^{3x}y\right)}{dx} = e^{4x}$$

Integrating we have

$$e^{3x}y = \frac{1}{4}e^{4x} + c$$

or

$$y = \frac{1}{4}e^x + ce^{-3x}$$

The initial condition implies

$$\frac{5}{4} = \frac{1}{4} + c$$

so $c = 1$ and

$$y = \frac{1}{4}e^x + e^{-3x}$$

3 [25 **points**]

$$\left(x^2 + x^2y^2\right)dx + e^{(x^3)}ydy = 0$$

Solution: This equation is separable, since it can be written as

$$x^2\left(1 + y^2\right)dx + e^{(x^3)}ydy = 0$$

or

$$x^2e^{-(x^3)}dx = -\frac{y}{1 + y^2}dy$$

Integrating we have

$$-\frac{1}{3}e^{-(x^3)} = -\frac{1}{2}\ln\left(1 + y^2\right) + c$$

or

$$3\ln\left(1 + y^2\right) - 2e^{-(x^3)} = k$$

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4 [25 **pts**.]

$$y' + \frac{4}{x}y = x^3y^3 \quad y(1) = 3$$

Solution: This is a Bernoulli equation. Multiplying both side of the equation by y^{-3} leads to

$$y^{-3}y' + \frac{4}{x}y^{-2} = x^3$$

Let $z = y^{-2}$. Then $z' = -2y^{-3}y'$ and we may rewrite the DE as

$$\frac{-1}{2}z' + \frac{4}{x}z = x^3$$

or

$$z' - \frac{8}{x}z = -2x^3$$

which is a first order linear DE in z. The integrating factor for this equation is

$$e^{-\int \frac{8}{x}dx} = e^{-8\ln x} = x^{-8}$$

Multiplying the DE for z by x^{-8} leads to

$$x^{-8}z' - 8x^{-9}z = -2x^{-5}$$

or

$$\frac{d\left(x^{-8}z\right)}{dx} = -2x^{-5}$$

or

$$x^{-8}z = -\frac{2}{-4}x^{-4} + c$$

Thus

$$z = \frac{1}{2}x^4 + cx^8$$

Since $z = y^{-1}$ we have

$$\frac{1}{y^2} = \frac{1}{2}x^4 + cx^8$$

The initial condition implies

$$\frac{1}{9} = \frac{1}{2} + c$$

or

$$c = -\frac{7}{18}$$

Finally we have

$$\frac{1}{y^2} = \frac{1}{2}x^4 - \frac{7}{18}x^8$$