Ma 221   Exam IIIA   12S

I pledge my honor that I have abided by the Stevens Honor System.   _____

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 _____

\#2 _____

\#3 _____

\#4 _____

Total Score   _____

Note: A table of Laplace Transforms is given at the end of the exam.

1 $\left(25 \text{ } \mathbf{pts.}\right)$ Use Laplace Transforms to solve the initial value problem

$$y'' + 6y' + 5y = 12e^t \quad y(0) = -1 \quad y'(0) = 7$$

2 $\left(20 \text{ pts.}\right)$ Find

$$\mathcal{L}^{-1}\left\{\frac{-5s-36}{(s+2)(s^2+9)}\right\}$$

3 $\left(30\ \textbf{pts.}\right)$ Find the series solution near $x = 0$ of the equation

$$(x^2 + 1)y'' + y = 0$$

Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first *six* nonzero terms of the solution.

4 $\left(25 \text{ pts}.\right)$ Find the eigenvalues, λ, and eigenfunctions for

$$y'' + \lambda y = 0; \quad y(0) = 0, \quad y'(1) = 0$$

Be sure to consider all values of λ.

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\dfrac{t^{n-1}}{(n-1)!}$	$\dfrac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\dfrac{1}{s-a}$		$s > a$
$\sin bt$	$\dfrac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\dfrac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		