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Lecture Section: $\qquad$

Ma 221
Exam IA Solutions
13S
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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem \#1 $\qquad$
$\qquad$
\#3 $\qquad$
\#4 $\qquad$

Total Score
$\qquad$

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Solve:
1 [25 pts.]

$$
\frac{d u}{d v}+\frac{\sec ^{2} v}{\tan v} u=\cot v \quad u\left(\frac{\pi}{4}\right)=\frac{\pi}{2}
$$

Solution: This equation if linear with $P(v)=\frac{\sec ^{2} v}{\tan v}$. The integrating factor is

$$
I=e^{\int P(v) d v}=e^{\int \frac{\sec ^{2} v}{\tan v} d v}=e^{\ln (\tan v)}=\tan v
$$

Multiplying the DE by this leads to

$$
\frac{d u}{d v} \tan v+u \sec ^{2} v=1
$$

or

$$
\frac{d}{d v}(u \tan v)=1
$$

Integrating we have

$$
u \tan v=v+C
$$

The initial condition implies

$$
u\left(\frac{\pi}{4}\right)=\frac{\pi}{4}+C=\frac{\pi}{2}
$$

Thus $C=\frac{\pi}{4}$ and the solution is

$$
u(v)=\left(v+\frac{\pi}{4}\right) \cot v
$$

2 [25 pts.]

$$
\frac{d y}{d x}=y \sqrt{x+2} ; \quad y(2)=1
$$

Solution: This equation is separable and may be written as

$$
\frac{d y}{y}=\sqrt{x+2} d x
$$

Thus

$$
\int \frac{d y}{y}=\int \sqrt{x+2} d x
$$

Integrating both sides leads to

$$
\ln |y|=\frac{2}{3}(x+2)^{\frac{3}{2}}+C
$$

Using the initial condition we get

$$
0=\frac{2}{3}(4)^{\frac{3}{2}}+C
$$

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so

$$
C=-\frac{16}{3}
$$

and the solution is

$$
\ln |y|=\frac{2}{3}(x+2)^{\frac{3}{2}}-\frac{16}{3}
$$

3 [25 pts.]

$$
\left(y e^{x y}+2 x y\right) d x+\left(x e^{x y}+x^{2}\right) d y=0
$$

Solution: Here

$$
M=y e^{x y}+2 x y, \quad N=x e^{x y}+x^{2}
$$

We check for exactness. Since

$$
M_{y}=N_{x}=x y e^{x y}+e^{x y}+2 x
$$

The equation is exact. Thus there exists a function $f(x, y)$ such that

$$
\begin{aligned}
& \frac{\partial f}{\partial x}=M \text { and } \frac{\partial f}{\partial y}=N \\
& \frac{\partial f}{\partial x}=M=y e^{x y}+2 x y
\end{aligned}
$$

Integrating with respect to $x$ while holding $y$ constant leads to

$$
f=e^{x y}+x^{2} y+g(y)
$$

where $g(y)$ is an unknown function of $y$. Now to find $g(y)$, note that $\frac{\partial f}{\partial y}=N$, that is

$$
\frac{\partial f}{\partial y}=x e^{x y}+x^{2}=x e^{x y}+x^{2}+g^{\prime}(y)
$$

So $g^{\prime}(y)=0$, and therefore $g(y)=C_{1}$ and the solution is given by

$$
f=e^{x y}+x^{2} y=C
$$

4 [25 pts.]

$$
y^{\prime}=5 y-5 t y^{3}
$$

Solution: We rewrite the equation as

$$
y^{\prime}-5 y=-5 t y^{3}
$$

This is a Bernoulli equation so we multiply by $y^{-3}$ and get

$$
y^{-3} y^{\prime}-5 y^{-2}=-5 t
$$

Let $z=y^{-2}$ so that $z^{\prime}=-2 y^{-3} y^{\prime}$. Then the DE becomes

$$
-\frac{1}{2} z^{\prime}-5 z=-5 t
$$

or
$\qquad$

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$$
z^{\prime}+10 z=10 t
$$

This is a first order linear DE with $P(t)=10$. The integrating factor is $e^{\int 10 d t}=e^{10 t}$. Multiplying the above DE by this we get

$$
e^{10 t_{Z}^{\prime}}+10 e^{10 t_{Z}}=10 t e^{10 t}
$$

which is equivalent to

$$
\frac{d\left(z e^{10 t}\right)}{d t}=10 t e^{10 t}
$$

Since $10 \int t e^{10 t} d t=\frac{1}{10} e^{10 t}(10 t-1)+C=t e^{10 t}-\frac{1}{10} e^{10 t}+C$ then

$$
z e^{10 t}=t e^{10 t}-\frac{1}{10} e^{10 t}+C
$$

or

$$
z=t-\frac{1}{10}+C e^{-10 t}
$$

and the solution is given by

$$
z=y^{-2}=t-\frac{1}{10}+C e^{-10 t}
$$

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## Table of Integrals

$$
\begin{array}{|l|}
\hline \int \frac{\sec ^{2} t}{\tan t} d t=\ln (\tan t)+C \\
\hline \int t e^{a t} d t=\frac{1}{a^{2}} e^{a t}(a t-1)+C \\
\hline \int t^{2} e^{a t} d t=\frac{1}{a^{3}} e^{a t}\left(a^{2} t^{2}-2 a t+2\right)+C \\
\hline \int \cos ^{2} t d t=\frac{1}{2} t+\frac{1}{4} \sin 2 t+C \\
\hline \int \cos ^{3} t d t=\frac{1}{3} \cos ^{2} t \sin t+\frac{2}{3} \sin t+C \\
\hline \int \sin ^{2} t d t=\frac{1}{2} t-\frac{1}{4} \pi-\frac{1}{4} \sin 2 t+C \\
\hline \int \sin ^{3} t d t=\frac{1}{12} \cos 3 t-\frac{3}{4} \cos t+C \\
\hline
\end{array}
$$

