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Ma 221 Exam IB Solutions
13S

Solve:

1 [25 **pts**.]

$$\frac{du}{dv} + (\tan v)u = \cos v \qquad u(\pi) = 2\pi$$

Solution: This equation if linear with $P(v) = \tan v$. The integrating factor is

$$I = e^{\int P(v)dv} = e^{\int \tan dv} = e^{\ln(\sec v)} = \sec v$$

Multiplying the DE by this leads to

$$\frac{du}{dv}\sec v + u \sec v \tan v = 1$$

or

$$\frac{d}{dv}(u \sec v) = 1$$

Integrating we have

$$\begin{aligned} u \sec v &= v + C \\ u &= v\cos v + C\cos v \\ &= (v + C)\cos v \end{aligned}$$

The initial condition implies

$$u(\pi) = (\pi + C)(-1) = 2\pi$$

Thus $C = -3\pi$ and the solution is

$$u(v) = (v - 3\pi)\cos v$$

2 [25 **pts**.]

$$\frac{dy}{dx} = (2x + 1)\sqrt{y}; \quad y(1) = 4$$

Solution: This equation is separable and may be written as

$$\frac{dy}{\sqrt{y}} = (2x + 1)dx$$

Thus

$$\int \frac{dy}{\sqrt{y}} = \int (2x + 1)dx$$

Integrating both sides leads to

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$$2\sqrt{y} = x^2 + x + C$$

Using the initial condition we get

$$4 = 2 + C$$

so

$$C = 2$$

and the implicit solution is

$$2\sqrt{y} = x^2 + x + 2$$

3 [25 **pts**.]

$$\left(ye^{xy} + y^2\right)dx + (xe^{xy} + 2xy)dy = 0$$

Solution: Here

$$M = ye^{xy} + y^2, \qquad N = xe^{xy} + 2xy$$

We check for exactness. Since

$$M_y = N_x = xye^{xy} + e^{xy} + 2y$$

The equation is exact. Thus there exists a function $f(x,y)$ such that

$$\frac{\partial f}{\partial x} = M \text{ and } \frac{\partial f}{\partial y} = N$$

$$\frac{\partial f}{\partial x} = M = ye^{xy} + y^2$$

Integrating with respect to x while holding y constant leads to

$$f = e^{xy} + xy^2 + g(y)$$

where $g(y)$ is an unknown function of y. Now to find $g(y)$, note that $\frac{\partial f}{\partial y} = N$, that is

$$\frac{\partial f}{\partial y} = xe^{xy} + 2xy = xe^{xy} + 2xy + g'(y)$$

So $g'(y) = 0$, and therefore we may choose $g(y) = 0$ and the solution is given by

$$f = e^{xy} + xy^2 = C$$

4 [25 **pts**.]

$$y' = 5y - 5ty^4$$

Solution: We rewrite the equation as

$$y' - 5y = -5ty^4$$

This is a Bernoulli equation so we multiply by y^{-4} and get

$$y^{-4}y' - 5y^{-3} = -5t$$

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Let $z = y^{-3}$ so that $z' = -3y^{-4}y'$. Then the DE becomes

$$-\frac{1}{3}z' - 5z = -5t$$

or

$$z' + 15z = 15t$$

This is a first order linear DE with $P(t) = 15$. The integrating factor is $e^{\int 15dt} = e^{15t}$. Multiplying the above DE by this we get

$$e^{15t}z' + 15e^{15t}z = 15te^{15t}$$

which is equivalent to

$$\frac{d\left(ze^{15t}\right)}{dt} = 15te^{15t}$$

Since $15\int te^{15t}dt = \frac{1}{15}e^{15t}(15t-1) + C = te^{15t} - \frac{1}{15}e^{15t} + C$ then

$$ze^{15t} = te^{15t} - \frac{1}{15}e^{15t} + C$$

or

$$z = t - \frac{1}{15} + Ce^{-15t}$$

and the implicit solution to the original d.e. is given by

$$z = y^{-3} = t - \frac{1}{15} + Ce^{-15t}$$

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Table of Integrals

$\int \frac{\sec^2 t}{\tan t} dt = \ln(\tan t) + C$
$\int te^{at} dt = \frac{1}{a^2} e^{at}(at - 1) + C$
$\int t^2 e^{at} dt = \frac{1}{a^3} e^{at}\left(a^2 t^2 - 2at + 2\right) + C$
$\int \cos^2 t dt = \frac{1}{2} t + \frac{1}{4} \sin 2t + C$
$\int \cos^3 t dt = \frac{1}{3} \cos^2 t \sin t + \frac{2}{3} \sin t + C$
$\int \sin^2 t dt = \frac{1}{2} t - \frac{1}{4}\pi - \frac{1}{4} \sin 2t + C$
$\int \sin^3 t dt = \frac{1}{12} \cos 3t - \frac{3}{4} \cos t + C$